J. Sci. HIROSHIMA UNIV. SER. A-I
34 (1970), 59-68

On KD-null Sets in N-dimensional Euclidean Space

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(Received February 24, 1970)

Introduction

Ahlfors and Beurling [1] introduced the notion of a null set of class N_D in the complex plane: A compact set E is a null set of class N_D if and only if every analytic function in $D(\Omega-E)$ can be extended to a function in $D(\Omega)$ for a domain Ω containing E, where $D(\Omega)$ is the class of single-valued analytic functions in Ω with finite Dirichlet integrals. They characterized a null set of class N_D by means of the span, the extremal length and the others. On the other hand, the class KD, which consists of all harmonic functions u with finite Dirichlet integrals such that $^*du$ is semiexact, was considered on Riemann surfaces and various characterizations of the class O_{KD} were given by many authors; see, for example, Rodin [5], Royden [7], Sario [8]. We can consider the class KD also on an N-dimensional euclidean space R^N ($N \geqq 3$) and define KD-null sets as a compact set E such that any function in $KD(\Omega-\mathrm{E})$ can be extended to a function in $KD(\Omega)$ for a bounded domain Ω containing E.

In the present paper, we shall prove some theorems on KD-null sets analogous to those on null sets of class N_D. In §3, we observe some relations between KD-null set and the span, which was introduced by Rodin and Sario [6] in Riemannian manifolds. Moreover we show that the N-dimensional Lebesgue measure of a KD-null set is equal to zero. In §4, we shall give a necessary condition for a set to be KD-null in terms of the extremal length.

The author is indebted to Professor M. Ohtsuka for his advice and encouragement.

§1. Preliminaries

We shall denote by $x=(x_1, x_2,\cdots, x_N)$ a point in R^N, and set $|x|=\sqrt{x_1^2+x_2^2+\cdots+x_N^2}$. By an unbounded domain in R^N we shall mean a domain which is equal to the complement of a compact set. A harmonic function u defined in an unbounded domain is called regular at infinity if $\lim_{|x|\to\infty} u(x)=0$. Consider a C^1-surface τ which divides R^N into a bounded domain and an unbounded domain. When we consider the normal derivative $\frac{\partial}{\partial n}$ at a point of τ, the normal is drawn in the direction of the unbounded domain.