J. Sci. HIROSHIMA UNIV. SER. A-I
34 (1970), 17-27

High Order Derivations II.

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(Received February 20, 1970)

This is a suite of the previous paper [3]. In that paper the senior author developed the fundamental calculus on high order derivations and proved some functorial properties of high order differentials. In this paper we shall apply these results to the theory of fields, in particular to a purely inseparable field extension of finite exponent. In §1 it will be shown that a purely inseparable extension of finite degree over a field K will be characterized by the fact that the derivation algebra $\mathcal{D}(L/K)$ coincides with the endomorphismring of L over K. If L is an extension of infinite degree over K this is not the case. But when L is of finite exponent over K we can introduce a suitable topology so as to get a bijective correspondence between the intermediate fields of L and K and the closed subrings of $\mathcal{D}(L/K)$ containing L. §3 is devoted to the representation theory of high order derivations. In the case of characteristic $p(>0)$ the high derivations of orders $1, p, p^2, \cdots$ are fundamental while in the case of characteristic zero every high order derivation can be represented as the sum of products of ordinary derivations.

Notations and terminologies: Let k and A be commutative rings such that A is a k-algebra and let M be an A-module. The set of q-th order derivations of A/k into M will be denoted by $\mathcal{D}_0^{(q)}(A/k, M)$. $\mathcal{D}_0^{(q)}(A/k, M)$ has a natural structure of left A-module. When $M=A$ we shall use the notation $\mathcal{D}_0^{(q)}(A/k)$ instead of $\mathcal{D}_0^{(q)}(A/k, A)$. We shall set $\mathcal{D}_0(A/k)=\bigcup_{q=1}^{\infty}\mathcal{D}_0^{(q)}(A/k)$. The derivation algebra $\mathcal{D}(A/k)$ is the direct sum of homotheties by elements of A and $\mathcal{D}_0(A/k)$, i.e., $\mathcal{D}(A/k)=A\oplus\mathcal{D}_0(A/k)$. $\mathcal{D}(A/k)$ is a subring of $\mathrm{Hom}_k(A, A)$. The module of q-th order differentials of A over k will be denoted by $\Omega_k^{(q)}(A)$ and the canonical q-th order derivation will be denoted by $\delta_{A/k}^{(q)}$. $\Omega_k^{(q)}(A)$ is a representing module for the functor $\mathcal{D}_0^{(q)}(A/k)$. Let B be an A-algebra. Then we have the canonical homomorphism $B\otimes_A\Omega_k^{(q)}(A)\rightarrow\Omega_k^{(q)}(B)$. The cokernel of this homomorphism will be denoted by $\Omega_k^{(q)}(B/A)$. The readers are expected to refer the paper [3] for details. In this paper we shall make frequent use of the results in [3] and the Proposition (or Theorem) 12 of Chapter I in [3], for example, will be quoted as I-12.

§1. Structure theorems for derivation algebras

Let k and A be commutative rings such that A is a k-algebra. Let φ be