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Functional Calculus in Locally Convex Algebras

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Introduction

L. Waelbroeck [16] and G.R. Allan [1] have shown that the contour integral technique is available in the case of locally convex algebras. Successively C.R. Ionescu-Tulcea [9] and F-Y. Maeda [11] considered operators in locally convex spaces which possess a functional calculus with functions in certain algebras containing analytic functions.

In the present paper we study the properties of elements in a locally convex algebra having a functional calculus with either analytic or $\mathscr{C}^{\infty}$-functions.

In §2 we give a perturbation formula generalizing a result contained in [3] (see also [4], II, Th. 1.5). In §3 we study the properties of elements which have a functional calculus by means of spectral distributions ([7]). We show that the regularity problem raised in [6], VI, 5(d) has a negative answer in the locally convex case (§4).

§ 1. Notations and preliminaries

Throughout, all linear structures are over the complex field Λ; Λ_∞ is the one-point compactification of Λ by ∞; R is the real field and N is the set of all natural numbers.

For any $\sigma\subset\Lambda$, $\sigma\neq\emptyset$, $0\leqq r<\infty$ we put

$$C(\sigma, r)=\{\lambda\in\Lambda;\ \mathrm{dist}\,(\lambda,\sigma)\leqq r\}.$$

If $\sigma=\emptyset$ then we put by definition $C(\emptyset, r)=\emptyset$, $0\leqq r<\infty$.

The closure in Λ (resp. Λ_∞) of a set σ is denoted by cl σ (resp. $\mathrm{cl}_\infty\sigma$).

If we put

$$D=\frac{1}{2}\left(\frac{\partial}{\partial\mathrm{Re}\lambda}+i\frac{\partial}{\partial\mathrm{Im}\lambda}\right),\qquad \bar{D}=\frac{1}{2}\left(\frac{\partial}{\partial\mathrm{Re}\lambda}-i\frac{\partial}{\partial\mathrm{Im}\lambda}\right)$$

then $\mathscr{C}^{\infty}$ denotes the algebra of all infinitely differentiable complex functions on Λ, endowed with the topology determined by the pseudonorms

$$\varphi\rightarrow|\varphi|_{n,K}=2^n\max_{j+k\leqq n}\sup_{\lambda\in K}|D^j\bar{D}^k\varphi(\lambda)|$$

for K compact and $n\in N$.