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On the Structure Space of a Direct Product of Rings

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§1. Introduction

It is known that, from the algebraic point of view, the ring E of entire functions has many interesting properties (see, for example, [3, §1, exerc. 12], [7] and [8]). Any residue ring $E/(f)$ by a non-zero entire function f is isomorphic to a direct product of homomorphic images of discrete valuation rings. This implies that, as far as the structure space is concerned, the study of the ring E is reduced to that of a direct product of discrete valuation rings. Thus, in this article, we shall mainly investigate the structure space of a direct product of commutative rings.

Every ring in this article will be assumed to be a commutative ring with an identity. In §2, as preliminaries, we shall give some relations between the structure space of a ring R, which will be denoted by Spec(R), and that of the Boolean algebra of idempotents in R. Next, in §3, we shall treat the case in which R is a direct product of local rings or integral domains; and in §5 the more restricted case, in which each factor of the product is a discrete valuation ring, will be treated by making use of some results on isolated subgroups of a totally ordered additive group which will be discussed in §4.

Finally, in §6, applying our theory to the ring of entire functions, we shall show how the algebraic properties of it, which was given by M. Henriksen, can be obtained (cf. [7], [8]).

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§2. Preliminaries

The set of idempotents in a ring R will be denoted by $B(R)$, or simply by B. The set $B(R)$ forms a Boolean algebra provided with the following order relation: for any x, y in B, $x \leq y$ if and only if $x = yx$. In this case the complement x' of x in B is $1-x$, $x \wedge y = xy$, $x \vee y = x + y - xy$, for any x, y in B.

The term "ideal" will be used with two meanings in this article. On the one hand, "ideal" will designate a ring ideal in a ring R. The word "ideal" will also be used to denote an ideal in a Boolean algebra $B(R)$, that is, a non-empty subset J of $B(R)$ such that $e \in J$, $f \in J$ implies $e \vee f \in J$, and $e \in J$, $f \leq e$ implies $f \in J$. Obviously, if A is an ideal in a ring R, then $A \cap B(R)$ is an