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On the Cohomology of Certain Quotient Manifolds of the Real Stiefel Manifolds and Their Applications

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§ 0. Introduction

Throughout this paper, p will denote an odd prime integer.

Let S^{2n+1} be the unit $(2n+1)$-sphere in the complex $(n+1)$-space. Then the free actions of $S^1=\{e^{i\theta}\mid 0\leq\theta<2\pi\}$ and $Z_p=\{e^{i\theta}\mid\theta=2\pi h/p,\ h=0,\cdots,p-1\}$ on S^{2n+1} are defined by $e^{i\theta}(z_0,\cdots,z_n)=(e^{i\theta}z_0,\cdots,e^{i\theta}z_n)$.

Let $V_{2n,k}$ be the Stiefel manifold of orthonormal k-frames in the real $2n$-space R^{2n}. We define free actions of S^1 and Z_p on $V_{2n,k}$ such that $e^{i\theta}$ operates on each vector of k-frame as above. We consider the quotient manifolds

$$Z_{2n,k}=V_{2n,k}/S^1, \qquad X_{2n,k}=V_{2n,k}/Z_p.$$

Then $Z_{2n,1}=CP^{n-1}$, the real $2n-2$ dimensional complex projective space, and $X_{2n,1}=L^{n-1}(p)$, the $2n-1$ dimensional mod p lens space.

Let ξ and η be the canonical complex line bundles over CP^∞ and $L^\infty(p)$, respectively. Then the above manifolds $Z_{2n,k}$ and $X_{2n,k}$ are homotopy equivalent to the total spaces of the associated $V_{2n,k}$-bundles of $n\xi$ and $n\eta$, respectively, as is shown in Proposition 1.3. Consequently, it is expected that the cohomology structures of $Z_{2n,k}$ and $X_{2n,k}$ give us the informations about the structures of $n\xi$ and $n\eta$ and so the immersion problem for lens spaces $L^n(p)$.

Recently, S. Gitler and D. Handel [5] have considered the projective Stiefel manifolds, which are the above manifolds $X_{n,k}$ for $p=2$ (in this case, n need not be even), and determined their mod 2 cohomology algebras and the actions of the Steenrod squares up to a small indeterminancy. Also, P. F. Baum and W. Browder [1] have determined completely the actions of the Steenrod squares when n is a power of 2. Moreover S. Gitler [6] has applied these results to the immersion problem for the real projective spaces.

The purpose of this paper is to study the mod p cohomology structures of $Z_{2n,k}$ and $X_{2n,k}$ and to apply these results to the problems of independent cross sections of $n\eta$ and immersions of $L^n(p)$.

In §1, we prove Theorem 1.11, which determines the mod p cohomology algebras $H^*(Z_{2n,k})$ and $H^*(X_{2n,k})$. Furthermore the generators are given in