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Monotone Limits in Linear Programming Problems

Maretsugu YAMASAKI

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§ 1. Introduction and problem setting

The aim of this paper is to investigate the behavior of values of linear programming problems under some monotone variations of objective functions and constraints.

More precisely, let X and Y be real linear spaces paired under the bilinear functional $((\ ,\ ))_1$, and let Z and W be real linear spaces paired under the bilinear functional $((\ ,\ ))_2$. A (linear) program for these paired spaces is a quintuple (A, P, Q, y_0, z_0). In this quintuple, A is a linear transformation from X into Z, P is a convex cone in X, Q is a convex cone in Z, y_0 is an element of Y and z_0 is an element of Z. The set S of feasible solutions for the program and the value M of the program are defined by

$$S=\{x \in P;\ Ax-z_0 \in Q\},$$

and

$$M=\inf \{((x,\ y_0))_1;\ x \in S\} \qquad \text{if} \quad S \neq \phi,$$

$$M=\infty \qquad \text{if} \quad S=\phi,$$

where ϕ denotes the empty set.

Let us denote the weak topology on X by $w(X, Y)$ and the Mackey topology on X by $s(X, Y)$ (cf. [2]). Let R be the set of real numbers and R_0 the set of non-negative real numbers. Let us define P^+ and Q^+ by

$$P^+=\{y \in Y;\ ((x,\ y))_1 \geqq 0 \qquad \text{for all} \quad x \in P\},$$

$$Q^+=\{w \in W;\ ((z,\ w))_2 \geqq 0 \qquad \text{for all} \quad z \in Q\}.$$

We say that the program (A, P, Q, y_0, z_0) is *regular* if A is $w(X, Y)-w(Z, W)$ continuous, P is $w(X, Y)$-closed and Q is $w(Z, W)$-closed.

We shall investigate some relations between the sequence $\{M_n\}$ of values of programs $(A_n, P_n, Q_n, y_n, z_n)$ and the value M of the program (A, P, Q, y_0, z_0) determined by any one of the following conditions:

(I) $A_n=A,\ y_n=y_0,\ z_n=z_0,$

(I. 1) $P_n \subset P_{n+1}$ and $P=\bigcup_{n=1}^{\infty} P_n,$