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Note on the Cauchy Problem for Linear Hyperbolic Partial Differential Equations with Constant Coefficients

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Let $P(D)$ be a linear partial differential operator of order $m \geq 1$ with constant coefficients, where D stands for $(D_0, D_1, \cdots, D_n)$, $D_0 = -i\dfrac{\partial}{\partial t}$, $D_1 = -i\dfrac{\partial}{\partial x_1}, \cdots, D_n = -i\dfrac{\partial}{\partial x_n}$. The Cauchy problem for $P(D)$ in $R^+_{n+1} = \{(t, x): t > 0\}$ and with initial hyperplane $t = 0$ will be understood in the sense of M. Itano [5]. If $P(D)$ is hyperbolic with respect to t-axis, the Cauchy problem to find $u \in \mathcal{D}'(R^+_{n+1})$ such that

$$P(D)u = f \qquad \text{in } R^+_{n+1}$$

with initial conditions

$$\lim_{t \downarrow 0} D_0^j u = \alpha_j \qquad j = 0, 1, \cdots, m-1,$$

for arbitrarily given $f \in \mathcal{D}'(R^+_{n+1})$ and $\alpha_j \in \mathcal{D}'(R_n)$, admits a unique solution u if and only if f has a canonical extension over $t = 0$. This follows from the hyperbolicity of $P(D)$ together with Corollary 1 in [5].

Our method of approach to study the problem will much rely upon the L^2-estimates, where $\mathcal{H}_{(m,s)}(R_{n+1})$ and the spaces related to it will play a central role. Strong hyperbolicity of $P(D)$ being not assumed, we can not make use of the energy inequality of Friedrichs-Levy's type in its own form. C. Peyser has derived an energy inequality from the properly hyperbolic operator [9]. On the other hand, recently S. L. Svensson has shown [10] that any hyperbolic operator is also properly hyperbolic in the sense of Peyser. Peyser considered the Cauchy problem only in the case of vanishing initial data, however, it will be possible to develop a more general treatment based on a modified energy inequality in which the initial data play a part. This will be done in this paper. By doing so, we have also succeeded in generalizing a result about a differential system established by J. Kopáček and M. Suchá [8] with a method of finite difference, and also succeeded in improving on some results of L. Hörmander [3, Theorem 5.6.4, p. 140] and A. Friedman [2, Theorem 14, p. 198] concerning the classical solutions.