HIROSHIMA MATH. J.
1 (1971), 349-372

On One-step Methods Utilizing the Second Derivative

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(Received September 20, 1971)

1. Introduction

Given a differential equation

$$y' = f(x, y) \tag{1.1}$$

and the initial condition $y(x_0) = y_0$, where the function

$$g(x, y) = f_x(x, y) + f(x, y) f_y(x, y) \tag{1.2}$$

is assumed to be sufficiently smooth. Let

$$x_i = x_0 + ih, \; y_i = y(x_i) \quad (i = 1, 2, \dots), \tag{1.3}$$

where h is a small increment in x and $y(x)$ is the solution to the given initial value problem. We are concerned with the case where the approximate values z_i of y_i $(i = 1, 2, \dots)$ are computed by means of the one-step methods, and put

$$T(x_0, y_0; h) = z_1 - y_1. \tag{1.4}$$

The one-step method of order p with μ stages for approximating y_1 can be expressed as follows:

$$z_1 = y_0 + h \sum_{i=1}^{\mu} q_i t_i, \tag{1.5}$$

where

$$T(x_0, y_0; h) = \boldsymbol{O}(h^{p+1}), \tag{1.6}$$

$$t_i = f(x_0 + a_i h, \; y_0 + h \sum_{j=1}^{\mu} b_{ij} t_j), \tag{1.7}$$

$$\sum_{j=1}^{\mu} b_{ij} = a_i \quad (i = 1, 2, \dots, \mu). \tag{1.8}$$

The method is called *explicit* when $b_{ij} = 0$ for $j \geqq i$. It is well known [2][1] that

1) Numbers in square brackets refer to the references listed at the end of this paper.