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K- and KO-Rings of the Lens Space $L^n(p^2)$ for Odd Prime p

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§1. Introduction

In the previous note [4], the structures of the K- and KO-rings of the standard lens space $L^n(4)=S^{2n+1}/Z_4$ are investigated, by considering the canonical complex line bundle and the non-trivial real line bundle over $L^n(4)$.

In this note, we shall study the $(2n+1)$-dimensional standard lens space mod p^r:

$$L^n(p^r)(=L^n(p^r;\,1,\cdots,1))=S^{2n+1}/Z_{p^r},$$

for prime p, by the similar methods to those which were used to determine the K- and KO-rings of $L^n(p)$ due to T. Kambe [3].

Let η be the canonical complex line bundle over $L^n(p^r)$, and

$$\sigma=\eta-1\in\tilde{K}(L^n(p^r)) \text{ and } \bar{\sigma}=r\sigma\in\widetilde{KO}(L^n(p^r))$$

be the stable class of η and the real restriction of σ. Then we have

THEOREM 1.1. (i) *Let p be a prime and $r\geqq 1$. Then, the order of the element σ^k of $\tilde{K}(L^n(p^r))$ is equal to p^{r+h}, $h=[(n-k)/(p-1)]$, for $1\leqq k\leqq n$; and $\sigma^{n+1}=0$.*

(ii) *Let p be an odd prime and $r\geqq 1$. Then, the order of the element $\bar{\sigma}^k$ of $\widetilde{KO}(L^n(p^r))$ is equal to $p^{r+h'}$, $h'=[(n-2k)/(p-1)]$, for $1\leqq k\leqq[n/2]$; and $\bar{\sigma}^{[n/2]+1}=0$.*

For the case $r=2$, the additive structures of $\tilde{K}(L^n(p^2))$ for prime p and $\widetilde{KO}(L^n(p^2))$ for odd prime p are determined as follows. Let

$$(1.2)\qquad n-p^i+1=a_i(p^{i+1}-p^i)+b_i \quad (0\leqq b_i<p^{i+1}-p^i) \quad \text{for } i=0,1,$$

and consider the following elements of $\tilde{K}(L^n(p^2))$:

$$\sigma=\eta-1,\quad \sigma(1)=\eta^p-1=(1+\sigma)^p-1,$$

$$(1.3)\qquad \sigma(1,k)=\begin{cases}\sigma(1)\sigma^k+p^{[(n-k)/p]}\sigma^{p+k} \\ \qquad\qquad (\text{if } b_1\leqq k<b_1+p-1 \text{ or } k<b_1-(p-1)^2) \\ \sigma(1)\sigma^k \quad (\text{otherwise}),\end{cases}$$