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# *Some Properties of Hopf Algebras Attached to Group Varieties*

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In the previous paper [8] we developed a theory of invariant semi-derivations on group varieties defined over an algebraically closed field $k$ of a positive characteristic $p$. Let $G$ be a group variety defined over $k$ and $\mathfrak{g}(G)$ the set of all left invariant semi-derivations of $G$. Then the direct sum $\mathfrak{H}(G)=k\oplus\mathfrak{g}(G)$ is a subalgebra of $\mathrm{End}_k(k(G))$, where $k(G)$ is the field of the ra-toinal functions on $G$ over $k$. This structure has a close connection with the group multiplication of $G$. On the other hand $\mathfrak{H}(G)$ may be identified with the set of point distributions of the local ring $\mathcal{O}$ of $G$ at the neutral element $e$, and then $\mathfrak{H}(G)$ has a structure of a coalgebra induced dually from that of $\mathcal{O}$ as an algebra over $k$. These structures give to $\mathfrak{H}(G)$ a Hopf algebra structre over $k$. Using this structure we obtained some results on purely inseparable isogenies of group varieties in [8].

In this paper we shall show that our theory of the Hopf algebras $\mathfrak{H}(G)$ has more applications not only to the theory of purely insparable isogenies of group varieties, but also to the general theory of algebraic groups over a field of a positive charabteristic $p$. In particular $\mathfrak{H}(G)$ may play a similar role to that of the Lie algebra of invariant derivations on a group variety in the case of characteristic zero.

In §1 we give some definitions and results on Hopf algebras over a field which are necessary in the later sections. Let $\mathcal{C}$ be the category of commutative and cocommutative Hopf algebras over a field $k$ which are a union of finite dimensional Hopf subalgebras. Then it is shown that $\mathcal{C}$ is an abelian category. In the next section we shall obtain a criterion, in the languages of Hopf algebras, for a morphism of a group variety to another to be separable. For this purpose we give a generalization of the theorem in the paper [4] on the existence of convenient pair of local parameters at the neutral elements for a given purely inseparable isogeny of group varieties. As an application of this criterion we give a modification of Serre's results on the group $\mathrm{Ext}(A, B)$ in §3, where $A$ and $B$ are commutative group varieties. He treated in [6] the case of purely inseparable isogenies of exponent 1 making use of the Galois theory for such isogenies. However we obtain the same result for any purely inseparble isogeny of a commutative group variety using our Hopf algebras. Of course this result may be obtained in a different way if we use the fact that the category of commutative algebraic group