HIROSHIMA MATH. J.
1 (1971), 171-176

Some Oscillation Criteria for nth Order Nonlinear Delay-Differential Equations

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(Received September 2, 1971)

1. Introduction.

Let us consider the nth order nonlinear delay-differential equation

$$(1)\qquad x^{(n)}(t)+\sum_{i=1}^{m} f_i(t)F_i(x_{d_{i,0}}(t),\ x'_{d_{i,1}}(t),\cdots,\ x^{(n-1)}_{d_{i,n-1}}(t))=0,$$

where

$$x^{(k)}_{d_{i,k}}(t)=x^{(k)}(t-d_{i,k}(t))$$

and the delays $d_{i,k}(t)$ are assumed to be continuous functions, nonnegative and bounded by some constant M on the half-line $[t_0,+\infty)$. In the special case where $d_{i,k}(t)=0$ for $i=1, 2,\cdots, m$, $k=0, 1,\cdots, n-1$, equation (1) clearly reduces to the ordinary differential equation

$$(2)\qquad x^{(n)}+\sum_{i=1}^{m} f_i(t)F_i(x,\ x',\cdots,\ x^{(n-1)})=0.$$

Let F be the family of solutions of (1) which are indefinitely continuable to the right. A solution $x(t)$ in F is said to be oscillatory if it has no last zero, i, e., if $x(t_1)=0$ for some t_1, then there exists some t_2, $t_2>t_1$, for which $x(t_2)=0$; otherwise a solution in F is nonoscillatory.

The purpose of this paper is to investigate the oscillatory properties of (1), giving sufficient conditions that all solutions of (1) in F are oscillatory in the case where n is even and are oscillatory or monotone in the case where n is odd. Our results generalize to arbitrary $n\geqq 2$ recent results of Staikos and Petsoulas [6] for the case $n=2$. It is to be noted that, still in the reduced case of the ordinary differential equation (2), our results improve previous results due to Kartsatos [1] and the present author [4], [5].

The author wishes to thank Professor T. Kusano for his interest in this work and for several helpful suggestions.

2. Oscillation Theorems.

We shall prove the following theorems.