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On the Radon Transform of the Rapidly Decreasing Functions on Symmetric Spaces II

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1. Introduction.

One of the problems which are proposed by S. Helgason for the Radon transform is to study the relations between the function spaces on a space X and on the dual space $\hat{X}$ by means of the Radon transform $f \to \hat{f}$. In [1], we considered the transform of the rapidly decreasing functions in $\mathcal{S}(S)$ on a Riemannian globally symmetric space S. But to construct a $\mathcal{S}$-theory for the Radon transform in a sense, it seems more favorable to study the Radon transform on the Schwartz space $\mathcal{C}(S)$, which is generalized by Harish-Chandra in [3], than on $\mathcal{S}(S)$, since we know that the Schwartz space is invariant under the left translations by G [3].

In this paper we shall study the Radon transform for the functions in the Schwartz space $\mathcal{C}(S)$ on a Riemannian globally symmetric space of the non-compact type. The main results are Theorems A, B, C and D.

2. Preliminaries.

As usual, $\boldsymbol{R}$ and $\boldsymbol{C}$ denote the fields of real and complex numbers respectively. If M and N are two topological spaces, φ a homeomorphism of M onto N and f a function on M, we put $f^{\varphi} = f \circ \varphi^{-1}$. If M is a C^{∞}-manifold, $C^{\infty}(M)$ (respectively, $C_c^{\infty}(M)$) denotes the space of differentiable functions (respectively, differentiable functions with compact support) on M. If G is a Lie group and K a closed subgroup of G, for $x \in G$, the left translation by x of the homogeneous space G/K of the left cosets onto itself will be denoted by $\tau(x)$.

$\boldsymbol{D}(G/K)$ denotes the algebra of differential operators on the homogeneous space G/K which are invariant under the left translations $\tau(x)$, $x \in G$. We write $\boldsymbol{D}(G)$ instead of $\boldsymbol{D}(G/e)$, where e is the identity element of G.

Let S be a Riemannian globally symmetric space of the noncompact type, and $G = I_0(S)$ denote the largest connected group of isometries of S in the compact open topology, then G is a semisimple Lie group and has no compact normal subgroup $\neq e$. Let any point o in S fix, K denote the isotropy subgroup of G at o, $\mathfrak{g}_0$ and $\mathfrak{k}_0$ denote the Lie algebras of G and K, respectively,