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Accretive Mappings in Banach Spaces

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Introduction

In the theory of semigroups of nonlinear contraction mappings, the notion of accretive mappings has appeared to be very practical (see [3], [6], [8]). In this paper, we study a multivalued accretive mapping A of a real Banach space X into itself. A is called *m-accretive* if the range of $I+A$ is the whole of X; thus it is useful in perturbation problems to know whether the given mapping is m-accretive.

It is known that if X is a Hilbert space, then an accretive mapping of X into itself is locally bounded at every point of the interior of its domain (see [10], [11]). We shall show that this is also true in case X is a reflexive Banach space provided that the duality mapping of X is bicontinuous (THEOREM 1), and use this fact to show that, under certain conditions, an accretive mapping is m-accretive if and only if it is maximal accretive (COROLLARY 1 of THEOREM 5).

In order to obtain the latter result, we consider the initial value problem of the evolution equation

(E) $$u'(t)+Au(t)\ni 0,\ u(0)=a.$$

This problem has a solution (in a certain sense) if A is m-accretive. However, it seems difficult to solve (E) without the m-accretiveness of A. It was shown in [7] that if X^* is uniformly convex and A is everywhere defined, singlevalued and hemicontinuous, then (E) has a global solution for any given $a\in X$ and A is m-accretive. We shall extend this result ot the case where A is multivalued, locally bounded, demiclosed and accretive (THEOREMS 4 and 5). As an application, we shall show that such a mapping A generates a nonlinear contraction semigroup on X (THEOREM 6).

§0. Definitions and notation

Throughout this paper let X be a real reflexive Banach space and X^* be the dual space. The natural pairing between $x\in X$ and $x^*\in X^*$ is denoted by $<x, x^*>$. The norms in X and X^* are denoted by $\|\cdot\|$. We denote by I the identity mapping of X onto X.

For a subset S of X, we denote by $\bar{S}$, $\mathring{S}$ and $co(S)$ the closure, the interior