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Oscillation Theorems for Delay Equations of Arbitrary Order

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1. Introduction

We consider the n-th order delay equations

$$x^{(n)}(t)+p(t)f(x(t),\ x(\delta(t)))=0, \tag{1}$$

$$x^{(n)}(t)+p(t)g(x(\delta(t)))=0, \tag{2}$$

where $p(t)$ is continuous and eventually positive on $R_{+}=[0, \infty)$ and $\delta(t)$ is continuous on R_{+} with $\delta(t)\leqq t$, $\lim_{t\to\infty}\delta(t)=\infty$. (These assumptions on $p(t)$ and $\delta(t)$ will be assumed without further mention.) We restrict attention to solutions of (1) or (2) which exist on some positive half-line. A nontrivial solution $x(t)$ is called oscillatory if there exists a sequence $\{t_k\}_{k=1}^{\infty}$ such that $\lim_{k\to\infty} t_k=\infty$ and $x(t_k)=0$ for all k. Otherwise, a solution is called nonoscillatory. A nonoscillatory solution is said to be strongly monotone if it tends monotonically to zero as $t\to\infty$ together with its first $n-1$ derivatives.

In [2] we established an oscillation theorem for (2) under the assumption that the retarded argument $\delta(t)$ is continuously differentiable and nondecreasing on R_{+}. The purpose here is to give oscillation criteria for (1) and (2) by avoiding this assumption and requiring that $\delta(t)$ has a continuously differentiable and nondecreasing minorant $\delta_*(t)$. The use of a differentiable minorant was suggested by Travis [4]. This will allow our theorems to be applied to delay equations of the form $x^{(n)}(t)+p(t)g(x(t-\tau(t)))=0$, $0\leqq\tau(t)\leqq M$, where $\tau(t)$ is not assumed differentiable.

2. Main Theorems

We now state our major results.

THEOREM 1. *With regard to equation* (1) *assume that*:

(i) *there exists a continuously differentiable and nondecreasing function on* R_{+}, $\delta_*(t)$, *such that* $\delta_*(t)\leqq\delta(t)$ *and* $\lim_{t\to\infty}\delta_*(t)=\infty$;

(ii) $f(x, y)$ *is continuous on* $R\times R$, $R=(-\infty, \infty)$, *is nondecreasing in* y,