HIROSHIMA MATH. J.
3 (1973), 121-151

Harmonic Functions on Real Hyperbolic Spaces

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(Received January 25, 1973)

Introduction

The present paper deals with the problem of an integral representation of harmonic functions of the laplacian on real hyperbolic spaces.

There are some types of theorems on Dirichlet problem, which show that certain classes of harmonic functions on the unit disc are given by the Poisson integral (cf. [1]). However, in order to obtain arbitrary harmonic functions we should consider the „Poisson transform of hyperfunctions", as S. Helgason showed in [6]. He proved there that any eigenfunction of the laplacian on the unit disc (with respect to the Poincaré metric) is given as the Poisson transform of a hyperfunction on the unit circle. In the case of Euclidean space [3], the situation is different. In this case we should consider a space which properly contains the hyperfunctions on the unit sphere.

It is our object to prove that, in the case of real hyperbolic spaces, the Poisson transform is an isomorphism of the space of hyperfunctions on the boundary onto the space of harmonic functions of the laplacian (Theorem 4.6 in § 4).

This paper consists of four sections. In § 1, we characterize the hyperfunctions on a compact real analytic riemannian manifold by their Fourier coefficients with respect to the eigenfunctions of the laplacian. In § 2, we show that any harmonic function on a symmetric space of rank one can be expanded in an absolutely convergent series of K-finite harmonic functions. In § 3 we restrict our argument to the case of real hyperbolic spaces and determine the K-finite harmonic functions by solving differential equations. In the final section we define the Poisson transform of hyperfunctions, and making use of the characterization of hyperfunctions, we prove Theorem 4.6.

§ 1. Hyperfunctions on a compact riemannian manifold.

In this section we characterize the hyperfunctions on a compact real analytic riemannian manifold by their Fourier coefficients with respect to the eigenfunctions of the laplacian on the manifold.

Let B be a compact real analytic manifold with a riemannian metric g, ω the laplacian corresponding to g and $L^2(B)$ the space of square integrable functions on B with respect to the measure induced by g. We denote the unitary inner