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The Paley-Wiener Theorem for Distributions on Symmetric Spaces

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1. Introduction

Let S be a symmetric space of the non-compact type. In his paper [10], S. Helgason obtained the Paley-Wiener theorem for the Fourier transform of the functions in $C_c^\infty(S)$.

The purpose of this paper is to characterize the Fourier transform of the distributions with compact support.

The crucial points of our proof are as follows. By means of convolution by a Dirac sequence we consider the regularization of tempered distributions (for the characterization of the Fourier transform of tempered distributions, see Theorem 6 in [1]) which, we notice, are the functions in $C_c^\infty(S)$. Then we use the above mentioned Helgason's Paley-Wiener theorem.

2. Notation and Preliminaries

As usual $\boldsymbol{R}$ and $\boldsymbol{C}$ denote the field of real numbers and the field of complex numbers, respectively. Let i denote a square root of -1. If M is a manifold, $C^\infty(M)$ and $C_c^\infty(M)$ denote the set of complex valued C^∞ functions on M and the set of C^∞ functions on M with compact support, respectively. If V is a finite dimensional vector space over $\boldsymbol{R}$, $\mathscr{S}(V)$ denotes the space of rapidly decreasing functions on V([12]) and $\boldsymbol{D}(V)$ denotes the algebra of differential operators with constant coefficients on V.

Let $\boldsymbol{R}^n$ be the n-dimensional Euclidean space, $|x|$ the Euclidean norm of $x \in \boldsymbol{R}^n$ and dx the Euclidean measure on $\boldsymbol{R}^n$. Let ρ be the function on $\boldsymbol{R}^n$ defined by

$$\rho(x)=\begin{cases} a\exp\left\{-\dfrac{1}{1-|x|^2}\right\} & \text{if } |x|<1, \\ 0 & \text{if } |x|\geqq 1, \end{cases}$$

where