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On a Class of Differential Operators with Polynomial Coefficients

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§1. Intoduction

In this paper, we study the existence and approximation of holomorphic solutions of a differential operator with polynomial coefficients. In general, we cannot expect the existence of holomorphic solutions even if the coefficients of an operator have no common zero ([7], [9], [10]). For example, in the complex two dimensional space $\mathbf{C}^2$, the equation

$$\left[x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}-1\right]u(x, y)=x$$

has no solution even in the space of formal power series.

An outline of this paper is as follows. In Section 2, we give some sufficient condition on a differential operator $L(\zeta, D)$ with polynomial coefficients under which $L(\zeta, D)\phi$ and ϕ have the same exponential type for every entire function ϕ (Theorem 1). This condition is then applied in Section 3 to show the existence and approximation of holomorphic solutions in some circular domain (Theorem 3).

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§2. Exponential type of entire solutions

Let $L(\zeta, D)$ be a differential operator with polynomial coefficients in $\mathbf{C}^n$. Then we can write

$$L(\zeta, D)=\sum_{\text{finite}} c_{\lambda\mu}\zeta^{\lambda}\left(\frac{\partial}{\partial\zeta}\right)^{\mu}, \tag{1}$$

where λ and μ are multi-indices, $c_{\lambda\mu}\in\mathbf{C}$, $\zeta^{\lambda}=\zeta_1^{\lambda_1}\cdots\zeta_n^{\lambda_n}$ and $\left(\frac{\partial}{\partial\zeta}\right)^{\mu}=\left(\frac{\partial}{\partial\zeta_1}\right)^{\mu_1}\cdots\left(\frac{\partial}{\partial\zeta_n}\right)^{\mu_n}$. We decompose L as follows:

$$L=L_l+L_{l+1}+\cdots+L_{l+k}, \qquad (k\geqq 0), \tag{2}$$

where $L_j=\sum_{|\lambda|-|\mu|=j} c_{\lambda\mu}\zeta^{\lambda}\left(\frac{\partial}{\partial\zeta}\right)^{\mu}$. We note that l may be a negative integer.