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Remarks on the m-Accretiveness of Nonlinear Operators

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Introduction

Let X be a real Banach space and let A be a multivalued operator from X into X, that is, to each $x \in X$ a subset Ax of X be assigned. We define $D(A)=\{x \in X;\ Ax \neq \phi\}$, $R(A)=\bigcup_{x \in X} Ax$ and $G(A)=\{[x, x'] \in X \times X;\ x' \in Ax\}$. We denote by F the *duality* mapping of X into the dual space X^*, i.e., it is defined by $Fx=\{x^* \in X^*;\ <x, x^*>=\|x\|^2=\|x^*\|^2\}$ for $x \in X$, where $< , >$ denotes the natural pairing between X and X^* and $\|\cdot\|$ denotes the norms in X and X^*. An operator A is called *accretive* in X, if for any $[x_i, x'_i] \in G(A)$, $i=1, 2$, there is an element $f \in F(x_1-x_2)$ such that $<x'_1-x'_2, f> \geq 0$, or equivalently,

$$\lim_{h \downarrow 0} \frac{1}{h}[\|x_1-x_2+h(x'_1-x'_2)\|-\|x_1-x_2\|] \geq 0 \tag{1}$$

(see R. H. Martin, Jr. [7]). An accretive operator A is called *m-accretive*, if $R(A+I)=X$.

It was shown in [6; THEOREM 1] that, under the uniform convexity of X^*, an accretive operator A is m-accretive if and only if it is demiclosed (i.e., for any sequence $\{[x_n, x'_n]\} \subset G(A)$, $x_n \to x$ strongly and $x'_n \to x'$ weakly in X imply that $[x, x'] \in G(A)$) and for each $z \in X$ and each $x \in D(A)$, the initial value problem: $u'(t)+Au(t)+z \ni 0$, $u(0)=x$ has a strong solution on $[0, \infty)$. In this note we do not require the uniform convexity of X^* and shall show an analogue of the above result in more general spaces, namely, in reflexive Banach spaces, by making use of the inequality (1) for accretiveness.

1. Main results

Let A be an operator from X into X and $\Omega=[0, r)$ or $[0, r]$ where $0<r \leq \infty$. Then an X-valued function u on Ω is called a *strong solution* of the initial value problem

$$u'(t)+Au(t) \ni 0, \qquad u(0)=a,$$

if $u(t)$ is strongly absolutely continuous on any bounded closed interval contained in Ω, $u(0)=a$ and the strong derivative $u'(t)$ exists, $u(t) \in D(A)$ and $u'(t)+Au(t) \ni 0$ for a.e. $t \in \Omega$. We denote by $\hat{D}(A)$ the set