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Weakly Coupled Parabolic Systems with Unbounded Coefficients

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Introduction

Consider the system of parabolic differential equations

$$(*)\qquad \sum_{i,j=1}^{n} a_{ij}^{\alpha}(x, t)\frac{\partial^2 u^\alpha}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b_i^\alpha(x, t)\frac{\partial u^\alpha}{\partial x_i} + \sum_{\beta=1}^{N} c^{\alpha\beta}(x, t)u^\beta - \frac{\partial u^\alpha}{\partial t} = 0,$$

$$\alpha = 1, 2, \dots, N.$$

Each equation of $(*)$ contains derivatives of just one component of the unknown functions $u^1(x, t)$, $u^2(x, t)$, ..., $u^N(x, t)$, and the system $(*)$ is coupled only in the terms which are not differentiated; so that a system of this form is said to be weakly coupled [11]. Such weakly coupled parabolic systems form a class to which most of the methods and techniques employed in the study of a single parabolic equation apply with minor necessary modifications.

During the last decade weakly coupled parabolic systems, both linear and nonlinear, have been intensively investigated by several authors, remarkably by Polish mathematicians. We refer in particular to the books of Protter and Weinberger [11], Szarski [12] and Walter [13], and the relevant references quoted in them.

The purpose of this paper is to add to the theory of weakly coupled parabolic systems results concerning the asymptotic behavior for $t \to \infty$ of solutions of the Cauchy problem for the system $(*)$ with unbounded coefficients. Specifically we focus our attention on the extension of the corresponding theorems which we have recently obtained for a single parabolic equation with unbounded coefficients [4]. Our results incidentally generalize those of one of the authors for more restricted classes of weakly coupled parabolic systems [7], [8].

The main tool is the maximum principle and the use of various comparison functions constructed so as to control the behavior of the solutions under consideration. The maximum principle is proved in § 1. The asymptotic behavior of solutions of $(*)$ is studied in §§ 2 and 3; § 2 concerns the exponential decay of the solutions with unbounded initial values, while § 3 concerns the exponential growth of the positive or negative solutions with nonvanishing initial values. In § 4 it

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