HIROSHIMA MATH. J.
3 (1973), 419–438

On the Cauchy Problem for Linear Hyperbolic Differential Equations with Multiple Characteristics and Constant Leading Coefficients

Kiyoshi YOSHIDA

(Received May 26, 1973)

In his paper [9, Theorem 1, p. 667] G. Peyser proved that when P is a hyperbolic differential polynomial with constant coefficients, $P+Q$ is always hyperbolic for an arbitrary differential polynomial Q with constant coefficients of order $<m-d$ $(0\leqq d<m)$ if and only if the degree of degeneracy of $P\leqq d$. Obviously P is strictly hyperbolic if and only if $d=0$. We can extend the result to the case where P is a hyperbolic differential polynomial with variable coefficients $\in\mathscr{B}(R_{n+1})$ having constant leading coefficients and Q is an arbitrary differential polynomial with variable coefficients $\in\mathscr{B}(R_{n+1})$.

The main purpose of this paper is to obtain some refinements of our previous paper [11] by taking into account the degree of degeneracy mentioned above. Our method of approaching the Cauchy problem relies largely upon the L^2-estimates as developed in [11]. Section 1 is devoted to the preliminary discussions by means of which our energy inequalities will be derived as shown in section 2. It is to be noticed that the energy inequalities obtained here coincide with those established in [11] provided that the degree of degeneracy in question equals $m-1$. After recalling the Cauchy problem taken in the sense of M. Itano [3] we shall establish in section 3 with the aid of the energy inequalities obtained above the uniqueness and the existence of the solutions, which present generalizations of the results in [11, Theorem 2.1, p. 453]. The final section deals with a generalization of G. Peyser's result.

1. Preliminaries

Let P be a differential polynomial in R_{n+1} written in the form $P=D_t^m+\sum_{\nu_0=0}^{m-1}\sum_{|\nu|\leqq m} a_\nu(t,x)D^\nu$, $a_\nu\in\mathscr{B}(R_{n+1})$, where $D=(D_t, D_x)$, $D_x=(D_1, D_2,\dots, D_n)$ with $D_t=\frac{1}{i}\frac{\partial}{\partial t}$, $D_j=\frac{1}{i}\frac{\partial}{\partial x_j}$, $j=1, 2,\dots, n$, and ν denotes a multi-index $\nu=(\nu_0, \nu')=(\nu_0, \nu_1,\dots,\nu_n)$. Throughout the present paper we shall assume that the principal part P_m of P has constant coefficients and that P is hyperbolic with respect to t, when each point (t, x) is fixed. For simplicity we shall call "hyperbolic" instead