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Remarks on Algebraic Hopf Subalgebras

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The aim of this note is to give a generalization of a theorem in the paper [2] which is concerned with algebraic Hopf subalgebras of the Hopf algebra attached to a group variety. In other words we show that a similar result to the theorem is obtained for not necessarily reduced group schemes over an algebraically closed field of a positive characteristic p, though the objects in [2] were group varieties exclusively. Moreover we give a corrected proof of Corollary to Lemma 12 in [2], because the previous proof is applicable only in the case where G is an affine algebraic group.

The terminologies are the same as in the papers [1] and [2].

1. In the following let k be an algebraically closed field of a positive characteristic p and G a group scheme of finite type over k. Let $\mathcal{O}=\mathcal{O}_{e,G}$ be the local ring of G at the neutral point e, that is, the stalk of the structure sheal of G at e. If $\mathcal{O}'$ is the local ring $\mathcal{O}_{e\times e,\, G\times G}$ of the product scheme $G\times G$ over k at the point $e\times e$, it is the quotient ring $(\mathcal{O}\otimes_k\mathcal{O})_S$ of $\mathcal{O}\otimes_k\mathcal{O}$ with respect to the multiplicatively closed set S which is the complement of the maximal ideal $\mathfrak{m}\otimes\mathcal{O}+\mathcal{O}\otimes\mathfrak{m}$ of $\mathcal{O}\otimes_k\mathcal{O}$, where $\mathfrak{m}$ is the maximal ideal of $\mathcal{O}$. Let R be the $\mathfrak{m}$-adic completion of $\mathcal{O}$. Then R has a natural structure of a formal group over k in the sense of §5 in [2], whose comultiplication Δ: $R\rightarrow R\hat{\otimes}_k R$ is given by the multiplication m of G. The antipode c of R is determined by the morphism $x\rightarrow x^{-1}$ of G to itself. Then R is called *the formalization of* G, and we remark that Proposition 7 of §5 in [2] is also true in this case. The proof is exactly the same.

First we give a corrected proof of the corollary to Lemma 12 in [2] in a slightly general form.

LEMMA 1. *Let G, $\mathcal{O}$ and $\mathcal{O}'$ be as above. Let $\mathfrak{a}$ be an ideal of $\mathcal{O}$ such that $\triangle(\mathfrak{a})\subset(\mathfrak{a}\otimes\mathcal{O}+\mathcal{O}\otimes\mathfrak{a})\mathcal{O}'$ and $c(\mathfrak{a})=\mathfrak{a}$. Let G' be the closed subset of G defined by the ideal $\mathfrak{a}$. Then G' is the underlying space of an irreducible group k-subscheme of G.*

PROOF. We may assume that $\mathfrak{a}$ is equal to its radical, because the radical of $\mathfrak{a}$ also satisfies the same hypothesis as $\mathfrak{a}$. From our assumption, it follows that there exists an open subset V of $G'\times G'$ containing $e\times e$ such that the image of V by the morphism m of $G\times G$ onto G is contained in G'. Since each irreducible