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# *Existence of Solutions of Heavily Nonlinear Volterra Integral Equations*

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## 1. Introduction

The objective of this paper is to show the existence of solutions (in a BANACH function space) of VOLTERRA integral equations of the form

$$x(t)=f(t)+\int_0^t K(t, s, x(s))ds, \tag{1.1}$$

where $x$, $f$, $K$ are $n$-dimensional vectors. To achieve this, we assume that "admissibility" conditions hold for a linear equation associated with (1.1). By "admissibility" we mean here the concept introduced by MILLER [12].

Our results are particularly useful in the case of equations of the form

$$x(t)=f(t)+\int_0^t K(t, s, x(s))x(s)ds, \tag{1.2}$$

(where $K$ is now an $n\times n$ matrix), provided that we know un upper bound for the norm of the linear operator $I-R_u$, where $I$ is the identity operator, and $R_u$ is the resolvent kernel associated with the linear equation

$$x(t)=f(t)+\int_0^t K(t, s, u(s))x(s)ds. \tag{1.2$_u$}$$

The function $u(t)$ above lies in a suitable closed ball of a Banach function space. We also show that the same method can be applied to nonlinear perturbations of linear systems.

## 2. Preliminaries

In what follows, $J=[0, \infty)$, $E=\{(t, s)\in J^2;\ t\geq s\}$, and $R=(-\infty, \infty)$. For a vector $x\in R^n$ we put $\|x\|=\sum_i|x_i|$, and for a real $n\times n$ matrix $A=[a_{ij}]$, $\|A\|=\sup_k\sum_i|a_{ik}|$. We denote by $C_c$ the space of all continuous functions $f$: $J\rightarrow R^n$, associated with the topology of uniform convergence on compact subintervals of $J$. The letter $B$ will always denote a BANACH space contained in $C_c$, stronger than $C_c$, and with norm $\|\cdot\|_B$. $C$ will stand for the space of all