HIROSHIMA MATH. J.
4 (1974), 99–132

(p, q)-Nuclear and (p, q)-Integral Operators

Ken-ichi MIYAZAKI

(Received September 17, 1973)

Recently, investigations have been made on the various generalizations of nuclear operators on the basis of the theory of locally convex topological vector spaces and the classes of operators in them ([2], [12], [13], [11], [8]). Among other things, p-absolutely summing operators due to A. Pietsch ([13]) and p-nuclear, p-integral, p-quasi-nuclear and p-quasi-integral operators due to A. Persson and A. Pietsch ([11]), $1 \leq p \leq \infty$, have played an important role in the study of classes of operators in connection with the classes of nuclear and integral operators in Banach spaces. Not only these operators were defined by making use of the norms of spaces L^p and l^p, but also their associated domains and ranges were closely related with spaces L^p and l^p ([13], [10], [11], [7], [8]). For instance, these operators were characterized with the aid of operators in L^p and l^p as follows ([11], [13]). A bounded linear operator T from a Banach space E to a Banach space F is p-nuclear (resp. p-integral, resp. p-absolutely summing) if and only if T can be factorized in the form $T=Q_1DP_1$ where $P_1 \in \boldsymbol{L}(E, l^\infty)$ with $\|P_1\| \leq 1$, $Q_1 \in \boldsymbol{L}(l^p, F)$ with $\|Q_1\| \leq 1$ and D is a multiplication operator by a sequence in l^p, (resp. if and only if T can be factorized in the form $T=Q_2IP_2$ where $P_2 \in \boldsymbol{L}(E, L^\infty)$ with $\|P_2\| \leq 1$, $Q_2 \in \boldsymbol{L}(L^p, F)$ with $\|Q_2\| \leq 1$ and I is the identity operator in $\boldsymbol{L}(L^\infty, L^p)$, resp. if and only if there exists a positive Radon measure μ on the weakly compact unit ball U° in E' such that $\|Tu\| \leq \rho\{\int_{U^\circ} |\langle u, u'\rangle|^p d\mu(u')\}^{1/p}$ for each $u \in E$ and with a positive constant ρ). With these in mind, by making use of Lorentz spaces $L^{p,q}$ and $l^{p,q}$ instead of L^p and l^p, the definitions and investigations of new classes of operators will be expected to be made. In the present paper, using the Lorentz spaces we shall introduce the four distinct types of operators, namely, the (p, q)-nuclear, (p, q)-integral, (p, q)-quasi-nuclear and (p, q)-quasi-integral operators, $1 \leq p, q \leq \infty$, which, in case $p=q$, coincide with the p-nuclear, p-integral, p-quasi-nuclear and p-quasi-integral operators respectively. The main purpose of this paper is to investigate these operators and to obtain their properties, their characterizations and the relationships among them. We also study the properties of the spaces of these operators with adequate quasi-norms. In these processes we shall be often concerned with Lorentz spaces $L^{p,q}$, $l^{p,q}$, where the notion and general properties of rearrangements of functions and of sequences are frequently used. Such utilizations of Lorentz spaces are of interest in themselves.

Section 1 is devoted to the preliminary remarks. We shall recall the defini-