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On the l-Number of 1-Cycles on an Abelian Variety

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Introduction

For any divisor D on an abelian variety X of dimension r, we denote by ϕ_D the homomorphism from X to the Picard variety $\hat{X}$ of X associated with the divisorial correspondence $m^*\mathcal{O}_X(D)\otimes p_1^*\mathcal{O}_X(-D)\otimes p_2^*\mathcal{O}_X(-D)$ on $X\times X$, where $m: X\times X\to X$ is the addition morphism of X and p_i: $X\times X\to X$ means the projection to the i-th component for $i=1, 2$. By *Riemann-Roch theorem*, the number of *Euler-Poincaré* $\chi(\mathcal{O}_X(D))$ is related with the self-intersection number (D^r) and with deg ϕ_D as follows;

$$\chi(\mathcal{O}_X(D))=\frac{(D^r)}{r!} \quad \text{and} \quad \deg\phi_D=\{\chi(\mathcal{O}_X(D))\}^2.$$

In particular, if D is ample, then $\chi(\mathcal{O}_X(D))=\dim \mathrm{H}^\circ(X, \mathcal{O}_X(D))=l(D)$, where $l(D)$ is the dimension of the linear system $L(D)$, and $\deg\phi_D$ becomes $l(D)^2$.

On the other hand, for 1-cycles on an abelian variety, an analogy to the relation $\deg\phi_D=1(D)^2$ was given by K. Toki [6], in the following way; that is, *for a positive* 1-*cycle C on an r-dimensional abelian variety X over the complex number fiield $\boldsymbol{C}$, the l-number $l(C)$, defined by $l(C)=\deg(\overset{r}{*}C)/r!$ where $\overset{r}{*}C$ means the r-times Pontrjagin product of C, satisfies the equality*

$$\deg\phi_C=l(C)^2.$$

Here ϕ_C is the homomorphism from Picard variety $\hat{X}$ of X to X defined by $\phi_C(\hat{x})=S(C\cdot(P|_{\{\hat{x}\}\times X}))$ for any $\hat{x}\in\hat{X}$, where P is the Poincaré divisor on $\hat{X}\times X$ and S means the sum on X. He proved this equality replacing the intersection and the Pontrjagin products by cohomological languages.

The purpose in this paper is to prove the same equality with no restriction on the characteristic of the ground field, by means of Jacobian varieties of curves.

In n°1, we shall review some results on abelian varieties and curves as preliminaries for our main theorem which will be proved in n°2.

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1. Let k be an algebraically closed field of any characteristic, which will be fixed throughout the paper. Let K, X and Y be three abelian varieties over k. If these abelian varieties form an exact sequence $0\longrightarrow K\xrightarrow{\iota} X\xrightarrow{\pi} Y\longrightarrow 0$ in the