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A Note on Hilbert's Nullstellensatz

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In his paper [3], S. Lang generalized the famous Hilbert's Nullstellensatz to the polynomial ring in an arbitrary number of variables over an algebraically closed field; however it seems to the author that his method is based on a usual technique known for the polynomial ring in a finite number of variables. Also, a number of proofs of Hilbert's Nullstellensatz have been given by O. Zariski and others ([1], [4], [5]). The main purpose of this note is to introduce the notion of the property $J(\Lambda)$ for a ring, which leads to a new approach to the theorem, applicable to the generalized case. We discuss, in 2, the relationship between Hilbert's Nullstellensatz and a Hilbert ring.

Throughout this note, a ring means a commutative ring with identity element.

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1. Let R be a ring. We denote by $Ht_1(R)$ the set of prime ideals of height 1 in R and for any given subset D of R, we denote by $H_R(D)$ the set of prime ideals of height 1 in R which contain at least one element of D. Let A be an R-algebra and Λ be a set. A is said to be Λ-generated over R if there is an R-algebra homomorphism from a polynomial ring $R[\dots, X_\lambda, \dots]$, $\lambda \in \Lambda$, onto A. In what follows the set Λ will always be assumed to be infinite.

If a subset D of R satisfies the following conditions: (1) $\mathrm{card}(D) \leq \mathrm{card}(\Lambda)$ and (2) any element of D is not a zero divisor, then we say that D is a J-subsetof R.

DEFINITION. When $H_R(D)$ is properly contained in $Ht_1(R)$ for any J-subset D, we say that the ring R has the property $J(\Lambda)$.

LEMMA 1. *Let R be a unique factrization domain such that the cardinality of the set of prime elements of R is greater than that of the set Λ. Then R has the property $J(\Lambda)$. In particular if k is a field such that card $(k) >$ card (Λ). then any polynomial ring over k has the property $J(\Lambda)$.*

The proof is almost clear and is omitted.

LEMMA 2. *Let $R \subseteq A$ be integral domains such that A is integral over R. Then if R has the property $J(\Lambda)$, then so does A.*

PROOF. Let $D = \{b_\mu; \mu \in M\}$ be any J-subset of A; let $f(X) = X^{n_\mu} + \cdots +$