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Integral Representations of Beppo Levi Functions of Higher Order

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Introduction

If f is a C^1-function with compact support on the Euclidean space R^n $(n \geqq 3)$, then it can be represented by its partial derivatives as follows:

$$f(x) = -\frac{1}{a_n} \sum_{i=1}^{n} \int \frac{\partial}{\partial t_i} |x-t|^{2-n} \frac{\partial f}{\partial t_i}(t) dt. \tag{1}$$

There are many ways to represent a C^m-function (m: positive integer) with compact support on R^n $(n \geqq 2)$ in terms of its partial derivatives of m-th order. Among them, the following two are regarded as generalizations of (1):

$$\varphi(x) = \sum_{|\alpha|=m} a_\alpha \int \frac{(x-y)^\alpha D^\alpha \varphi(y)}{|x-y|^n} dy \tag{2}$$

(Yu. G. Reshetnyak [9]), and

$$\varphi(x) = \begin{cases} \sum_{|\alpha|=m} c_\alpha \int D^\alpha(|x-y|^{2m-n}) D^\alpha \varphi(y) dy \\ \qquad \text{if } n-2m>0 \text{ or } n \text{ is odd} \\ \qquad \text{and } n-2m<0, \\ \sum_{|\alpha|=m} c'_\alpha \int D^\alpha(|x-y|^{2m-n} \log|x-y|) D^\alpha \varphi(y) dy \\ \qquad \text{if } n \text{ is even and } n-2m \leqq 0 \end{cases} \tag{3}$$

(H. Wallin [11]).

On the other hand, J. Deny and J. L. Lions [5] studied the space of Beppo Levi functions, e.g., the space $BL(L^p(R^n))$ of distributions on R^n whose partial derivatives belong to $L^p(R^n)$. They showed that any quasi continuous function f in $BL(L^2(R^n))$ $(n \geqq 3)$ is represented as (1) quasi everywhere, with an additional constant. M. Ohtsuka [8] extended their results to p-precise functions, which belong to $BL(L^p(R^3))$, and gave many other properties of precise functions in his lectures at Hiroshima University.

In this paper, we consider the space $BL_m(L^p(R^n))$ of Beppo Levi functions