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On the Stability of Finite-difference Schemes of Lax-Wendroff Type

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1. Introduction

Let us consider the initial value problem for a linear hyperbolic system

$$\frac{\partial u}{\partial t}=\sum_{j=1}^{n}A_j\frac{\partial u}{\partial x_j}\qquad(-\infty<x_j<\infty,\ 0\leqq t\leqq T), \tag{1.1}$$

$$u(x,0)=u_0(x), \tag{1.2}$$

where u is an N-vector function of the real variables $x=(x_1, x_2,\dots, x_n)$ and t, $A_j(j=1, 2,\dots, n)$ are real constant $N\times N$ matrices, and $u_0(x)$ is a vector function belonging to L_2. It is assumed that the solution to this initial value problem exists and is unique.

For the numerical solution of this problem we use the finite-difference schemes of Lax-Wendroff type. Several sufficient conditions for their stability in the sense of Lax-Richtmyer [4][1] are obtained when (1.1) is a symmetric hyperbolic system [4, 3, 2] and when it is a strictly hyperbolic system [5]. The object of this paper is to obtain some sufficient conditions for stability when (1.1) is a strongly hyperbolic system.

2. Notations and preliminaries

We denote by $|y|$ the Euclidean norm of the vector $y=(y_1, y_2,\dots, y_n)$, also denote by $|A|$ the spectral norm of the matrix A and put

$$A(y)=\sum_{j=1}^{n}A_jy_j,\qquad A_0(y)=A\left(\frac{y}{|y|}\right)\qquad(y\neq 0). \tag{2.1}$$

In the sequel we assume that the eigenvalues of $A_0(y)$ are all real for any real $y\neq 0$ and that there exist a non-singular matrix $T(y)$ and a constant C_1 independent of y such that

$$T(y)A_0(y)T(y)^{-1}=D_0(y), \tag{2.2}$$

1) Numbers in square brackets refer to the references listed at the end of this paper.