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# *Orbit Method and Nondegenerate Series*

Joseph A. WOLF

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**1.** If $G$ is a reductive Lie group, then its Plancherel formula ([1], [2], [8]) involves a series of representations for each conjugacy class of Cartan subgroups. These "nondegenerate series" are realized [8] by the action of $G$ on square integrable cohomology of partially holomorphic vector bundles over certain $G$-orbits on complex flag manifolds. That is similar to their realization by the Kostant-Kirillov orbit method using semisimple orbits. The differences occur when $G$ has noncommutative Cartan subgroups, and also for representations with singular infinitesimal character, i.e. when the semisimple orbit is not regular. Recently Wakimoto [6] used possibly-nonsemisimple orbits to realize the principal series, which is the series for a maximally noncompact Cartan subgroup $H$, when $G$ is a connected semisimple group and $H$ is commutative (e.g. when $G$ is linear). Here we use our method [8] to extend Wakimoto's procedure and realize all but a few members of every nondegenerate series of unitary representation classes for a reductive group. In the case of regular infinitesimal character there is no essential change from [8]. But in the case of singular infinitesimal character we rely on results of Ozeki and Wakimoto ([4], [6]), using nonsemisimple orbits in an interesting way.

To avoid repetition we assume some acquaintance with [8].

**2.** $G$ will be a reductive Lie group of the class studied in [8] and [9]. Thus its Lie algebra

(2.1a) $\mathfrak{g}=\mathfrak{c}+\mathfrak{g}_1$ with $\mathfrak{c}$ central and $\mathfrak{g}_1 = [\mathfrak{g}, \mathfrak{g}]$ semisimple,

we assume

(2.1b) if $g \in G$ then $Ad(g)$ is an inner automorphism on $\mathfrak{g}_C$,

and we suppose that $G$ has a closed normal abelian subgroup $Z$ such that

(2.2a) $Z$ centralizes the identity component $G_0$ of $G$,

(2.2b) $ZG_0$ has finite index in $G$, and

(2.2c) $Z \cap G_0$ is co-compact in the center $Z_{G_0}$ of $G_0$.

Then the adjoint representation maps $G$ to a closed subgroup $\bar{G}=G/Z_G(G_0)$ of

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