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On the Oscillation of Second Order Nonlinear Ordinary Differential Equations

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Introduction

In this paper we are concerned with the oscillatory behavior of solutions of the second order nonlinear differential equation

$$(r(t)x')' + a(t)f(x) = 0, \tag{A}$$

where the following assumptions are assumed to hold:

(a) $a \in C[0, +\infty)$;
(b) $r \in C^1[0, +\infty)$, and $r(t) > 0$ for $t \geqq 0$;
(c) $f \in C(-\infty, +\infty) \cap C^1(-\infty, 0) \cap C^1(0, +\infty)$, $\operatorname{sgn} f(x) = \operatorname{sgn} x$, and $f'(x) \geqq 0$ for $x \neq 0$.

We restrict our attention to solutions of (A) which exist on some half-line $[t_0, +\infty)$, where t_0 may depend on the particular solution. Such a solution is called oscillatory if it has arbitrarily large zeros; otherwise, a solution is called nonoscillatory.

The problem of determining if all solutions of equation (A) are oscillatory has been the subject of intensive investigations since the pioneering work of Atkinson [1], and during the last decade an extensive amount of study has been devoted to obtain sufficient conditions for oscillation of all solutions of (A) when the coefficient $a(t)$ is allowed to assume negative values for arbitrarily large values of t. For results on the subject we cite those given in the papers [2–5, 7–13, 15–20] as being representative. In particular, we refer the reader to a general oscillation theorem of Kamenev [8] which yields as particular cases oscillation criteria of Kiguradze [12], Kamenev [7] and others.

The purpose of this paper is to derive from the above mentioned theorem of Kamenev and its variant several criteria for oscillation of all solutions (or all bounded solutions) of the damped differential equation

$$x'' + q(t)x' + p(t)f(x) = 0. \tag{B}$$

Our results include an improvement and an extension of some of the recent results of Erbe [6] and Naito [14] for equation (B). Our approach seems natural as