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*Asymptotic Behavior of Solutions for Large $|x|$ of Weakly Coupled Parabolic Systems with Unbounded Coefficients**)

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§ 1. Introduction.

Let E^n be the n-dimensional Euclidean space whose points x is represented by its coordinates $(x_1,\dots, x_n)$. The distance of a point x of E^n to the origin is defined by $|x|=\left(\sum_{i=1}^{n} x_i^2\right)^{\frac{1}{2}}$. Every point in $D\equiv E^n\times(0, T]$ is denoted by (x, t), $x\in E^n$, $t\in(0, T]$ $(T<+\infty)$.

We say that a function $w(x, t)$ belongs to class $E_{\lambda\mu}(D, M, k)$ or shortly $E_{\lambda\mu}$ ($\lambda, \mu>0$ are constants) in D if there exist positive numbers M, k such that

$$|w(x, t)| \leqslant M\exp\{k[\log(|x|^2+1)+1]^{\lambda}(|x|^2+1)^{\mu}\}.$$

We say that a function $w(x, t)$ belongs to class $E_\lambda(D, M, k)$ or shortly E_λ ($\lambda\geqslant 1$ is a constant) in D if there exist positive numbers M, k such that

$$|w(x, t)| \leqslant M\exp\{k[\log(|x|^2+1)+1]^{\lambda}\}.$$

Consider a weakly coupled parabolic system of the form

$$(*)\quad F^p[u^p]\equiv \sum_{i,j=1}^{n} a_{ij}^p(x, t)\frac{\partial^2 u^p}{\partial x_i\partial x_j}+\sum_{i=1}^{n} b_i^p(x, t)\frac{\partial u^p}{\partial x_i}+\sum_{q=1}^{N} c^{pq}(x, t)u^q-\frac{\partial u^p}{\partial t}$$

$$p=1,\dots, N$$

with variable coefficients $a_{ij}^p(=a_{ji}^p)$, b_i^p, c^{pq} defined in $\bar{D}$.

In this paper, we deal with the decay of solutions of

$$(1)\qquad F^p[u^p]=0,\qquad p=1,\dots, N,$$

and the growth of solutions of

$$(2)\qquad F^p[u^p]\leqslant 0,\qquad p=1,\dots, N,$$

for large $|x|$.

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