Hiroshima Math. J.
5 (1975), 127–134

Conjugates of (p, q; r)-Absolutely Summing Operators

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(Received September 12, 1974)

§ 1. Introduction

By K. Miyazaki [4] a linear operator T from a Banach space E into another Banach space F is said to be $(p, q; r)$-absolutely summing for $1 \leq p, q, r \leq \infty$ if there exists a constant c such that for every finite sequence $\{x_i\}$ in E the inequality

$$\{\sum_i (i^{1/p-1/q}\| Tx_i\|^*)^q\}^{1/q} \leq c \sup_{\|x'\|\leq 1} (\sum_i |<x_i, x'>|^r)^{1/r}$$

is satisfied. Here $\{\| Tx_i\|^*\}$ denotes the non-increasing rearrangement of $\{\| Tx_i\|\}$, and as usual $\{\sum_i(\dots)^q\}^{1/q}$ and $(\sum_i|\dots|^r)^{1/r}$ are supposed to mean sup for $q=\infty$ and $r=\infty$ respectively. Especially, $(p, p; r)$-absolutely summing operators are exactly (p, r)-absolutely summing operators which were defined by B. Mitjagin and A. Pełczyński [3] and $(p, p; p)$-absolutely summing operators coincide with absolutely p-summing operators which are due to A. Pietsch [6]. The conjugates of absolutely p-summing operators have been investigated by several authors and especially characterized by J. S. Cohen [1] as strongly p'-summing operators where $1/p+1/p'=1$. The purpose of this paper is to investigate the conjugates of $(p, q; r)$-absolutely summing operators.

We shall introduce the notion of $(r; p, q)$-strongly summing operators and show that the conjugates of $(p, q; r)$-absolutely summing operators are $(r'; p', q')$-strongly summing operators where $1/p+1/p'=1/q+1/q'=1/r+1/r'=1$ and that the converse holds under a certain assumption. As a consequence of this result, we shall characterize the conjugates of (p, q)-absolutely summing operators.

The author would like to thank Professor S. Tôgô for his valuable comments in preparing this paper.

§ 2. Conjugates of (p, q; r)-absolutely summing operators

Let E and F be Banach spaces and let E' and F' be their continuous dual spaces. Let K be the real or complex field.

For $1 \leq p \leq \infty$ a sequence $\{x_i\}$ with values in E is called weakly p-summable provided for any $x' \in E'$ the sequence $\{<x_i, x'>\}$ belongs to l_p. The space $l_p(E)$ of weakly p-summable sequences is a normed space with the norm