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Nonoscillation Criteria for Fourth Order Elliptic Equations

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The problem of oscillation and nonoscillation of solutions of elliptic partial differential equations has been the subject of numerous investigations. For nonoscillation results we refer to Headley [2], Headley and Swanson [3], Kreith [4], Kuks [5], Piepenbrink [6], Skorobogat'ko [7], Swanson [8] and Yoshida [9]. All of these papers deal with second order elliptic equations or systems, and the author knows of no nonoscillation criteria which are applicable to equations of higher order.

Our purpose here is to develop nonoscillation criteria for the fourth order elliptic equation with real coefficients

$$
\begin{aligned}
(1)\qquad Lu = &\sum_{i,j,k,l=1}^{n} D_{ij}(\alpha_{ij}(x)\alpha_{kl}(x)D_{kl}u) + 2\beta(x)\sum_{k,l=1}^{n}\alpha_{kl}(x)D_{kl}u \\
&+ \sum_{i,j=1}^{n} D_i(a_{ij}(x)D_ju) + 2\sum_{i=1}^{n} b_i(x)D_iu + c(x)u = 0
\end{aligned}
$$

defined in an unbounded domain R of Euclidean n-space E^n. As usual, points in E^n will be denoted by $x=(x_1,\dots, x_n)$, differentiation with respect to x_i by D_i, $i=1,\dots, n$, and successive differentiation with respect to x_i and x_j by D_{ij}, $i, j= 1,\dots, n$. The following assumptions will be made throughout:

(a) The coefficients $\alpha_{ij}\in C^2(R)$, $\beta\in C(R)$, $a_{ij}\in C^1(R)$, $b_i\in C^1(R)$ and $c\in C(R)$.
(b) The matrix (α_{ij}) is symmetric and positive definite in R.
(c) The matrix (a_{ij}) is symmetric and negative semidefinite in R.

These assumptions will be placed without further mention on the coefficients of elliptic operators of the same form as L which will be considered in the sequel.

The domain $\mathfrak{D}(L; G)$ of L relative to any subdomain G of R is defined as the set $C^4(G)\cap C^2(\bar{G})$. The notation

$$R_r = R\cap \{x\in E^n: |x| > r\},\qquad 0<r<\infty,$$

will be used throughout.

DEFINITION 1. A bounded subdomain G of R is called a *nodal* domain of (1) if there exists a nontrivial solution $u\in\mathfrak{D}(L; G)$ of (1) such that $u=D_iu=0$