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A Note on Coreflexive Coalgebras

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Introduction

E. J. Taft [6] has introduced the concept of coreflexive coalgebras. Finite-dimensional coalgebras are coreflexive and the coalgebra of divided powers is coreflexive. The latter is a cocommutative coconnected coalgebra and its space of primitive elements is 1-dimensional. Taft has shown that if a cocommutative coconnected coalgebra is coreflexive, then the space of primitive elements is finite-dimensional. In this paper we show the converse of this result.

To this end, following D. E. Radford's idea in discussing coreflexivity in [3], we introduce a topology in the dual algebra of a coalgebra and give a necessary and sufficient condition for a coalgebra to be coreflexive.

Throughout this paper we employ the notations and terminology used in [4] and [6]. All vector spaces are over a fixed field k. For a vector space V and a subspace X of V

$$X^{\perp} = \{v^* \in V^*: \langle v^*, X\rangle = 0\}$$

and for a subspace Y of V^*

$$Y^{\perp} = \{v \in V: \langle Y, v\rangle = 0\}.$$

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1. The following lemma was indicated in [4], p. 240.

LEMMA 1. *Let $\{C_\mu, \sigma^\mu_\nu\}$ be an inductive system with a directed set M. If every C_μ has a coalgebra structure and every σ^μ_ν is a coalgebra map, then $C = \varinjlim C_\mu$ has a coalgebra structure such that every canonical map σ^μ: $C_\mu \to C$ is a coalgebra map.*

Furthermore, the dual algebra C^* is isomorphic to $\varprojlim C^*_\mu$ as algebras by the canonical map.

PROOF. We denote by Δ_μ and ε_μ the coalgebra structure of C_μ. Since σ^μ_ν is a coalgebra map the maps Δ_μ induce a map Δ': $C \to \varinjlim (C_\mu \otimes C_\mu)$ such that