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Codivisorial and Divisorial Modules over Completely Integrally Closed Domains (I)

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Introduction

Krull domains are, roughly speaking, congruent to Dedekind domains modulo the primes of height $\geqq 2$. This principle has brought on many results on Krull domains, which generalize the corresponding ones on Dedekind domains; for example, the fundamental theorem on ideals in a Dedekind domain can be formulated for a Krull domain as follows: For an ideal $\mathfrak{a}$ of a Krull domain A, there are primes $\mathfrak{p}_1, \dots, \mathfrak{p}_r$ of height 1, which are uniquely determined, so that $A:(A;\mathfrak{a}) = A:(A:\mathfrak{p}_1 \dots \mathfrak{p}_r)$; here the operation $A:(A:*)$ corresponds to the modulus "the primes of height $\geqq 2$".

It is well known that the notion of divisorial ideals plays an important role in the theory of Krull domains; in fact, the divisorial ideals represent the quotient of the set of ideals modulo the primes of height $\geqq 2$. However it seems to the authors that the importance of the notion of divisorial modules, which generalizes that of divisorial ideals, has not been recognized yet except the case of lattices.

The purpose of this paper is to introduce the notion of divisorial modules over a completely integrally closed domain by means of codivisorial modules and also to develope a theory of them. The key theorem is Theorem 1 (§1) which is valid for a completely integrally closed domain; this is the reason why we are mainly concerned with modules over completely integrally closed domains rather than Krull domains. In §2, we study modules over Krull domains exclusively.

§1. Codivisorial and divisorial modules over a completely integrally closed domain.

1. Let A be a completely integrally closed domain and K be its quotient field. We say that a fractional ideal $\mathfrak{a}$ of A is divisorial if $\mathfrak{a}$ is an intersection of principal ideals. It is well known that, for a fractional ideal $\mathfrak{a}$, $A:(A:\mathfrak{a})$ is the intersection of principal ideals which contain $\mathfrak{a}$ and is the smallest divisorial ideal containing $\mathfrak{a}$; we denote by $\tilde{\mathfrak{a}}$ the ideal $A:(A:\mathfrak{a})$. For fractional ideals $\mathfrak{a}$ and $\mathfrak{b}$, we say that $\mathfrak{a}$ is equivalent to $\mathfrak{b}$ if $\tilde{\mathfrak{a}} = \tilde{\mathfrak{b}}$; this relation is an equivalence relation and is denoted by $\sim$. The set of divisorial ideals can be identified with the quotient