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S^3 Actions on 4 Dimensional Cohomology Complex Projective Spaces

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§1. Introduction

Recently, F. Uchida [5] has determined smooth $SU(3)$ actions on homotopy complex projective spaces $hP_3(C)$.

The purpose of this note is to study smooth S^3 $(=SU(2))$ actions on cohomology complex projective planes by the analogous methods.

Let C and H be the complex and quaternion fields. Regard the complex projective plane as

$$P_2(C) = P(H \times C)$$

by the right complex multiplication. Then the smooth S^3 $(\subset H)$ action on $P_2(C)$ is given by

$$q\cdot[p, a] = [qp, a] \qquad (q \in S^3, p \in H, a \in C). \tag{1.1}$$

Also, regard H as the right complex vector space, set

$$P_2(C) = P(C^3) = P(H \otimes_C H/\sim)$$

where $p \otimes q \sim q \otimes p$ $(p, q \in H)$, and consider the smooth S^3 action on $P_2(C)$ given by

$$r\cdot[p \otimes q] = [rp \otimes rq] \qquad (r \in S^3, p, q \in H). \tag{1.2}$$

Now consider a 4 dimensional orientable closed smooth manifold

$$M = CHP_2(C),$$

having the same cohomology ring as $P_2(C)$, and assume that M admits a non-trivial smooth S^3 action.

Then, we obtain the following main theorem.

THEOREM 1.3. *If M satisfies the above conditions, then M is S^3 equivariantly diffeomorphic to the complex projective plane $P_2(C)$ with the S^3 action given by* (1.1) *or* (1.2). *In each case, the principal isotropy subgroup is the unit group $\{1\}$ or the cyclic group Z_4 of order* 4, *and the fixed point set $F(S^3, M)$ consists of*