HIROSHIMA MATH. J.
**5** (1975), 215–222

# *Note on Self-Maps Inducing the Identity Automorphisms of Homotopy Groups*

Kouzou TSUKIYAMA

(Received January 16, 1975)

## §1 Introduction

Let $X$ be a connected $CW$-complex with the base point $*$. Then we can consider the group $\mathscr{E}(X)$ of all based homotopy classes of self homotopy equivalences of $(X, *)$. This group has been studied by several authors.

Also, we can consider the subgroup

$$\mathscr{E}_{\#}(X) \qquad (\subset \mathscr{E}(X)), \tag{1.1}$$

formed by self-maps of $(X, *)$ inducing the identity automorphisms of all homotopy groups $\pi_i(X)$. Then $\mathscr{E}_{\#}(X)=1$ means that a (continuous) map $\psi\colon (X, *)\to (X, *)$ is homotopic rel $*$ to the identity map if and only if $\psi$ induces the identity automorphisms, that is,

$$\psi_* = \mathrm{id}\colon \pi_i(X) \longrightarrow \pi_i(X) \qquad \text{for all} \quad i.$$

The purpose of this note is to study a sufficient condition for $\mathscr{E}_{\#}(X)=1$.

Let $\{X_n | n \geqq 1\}$ be a Postnikov system of $X$, that is, $X_n$ be a $CW$-complex obtained by attaching $(i+1)$-cells $(i>n)$ to $X$ so that $X_n$ kills the homotopy groups $\pi_i(X)$ for $i>n$. Then we can consider the cohomology group

$$H^n(X_{n-1}; \pi_n(X)), \text{ with the local coefficient,} \tag{1.2}$$

where $\pi_1(X_{n-1}) \cong \pi_1(X)$ acts on the coefficient $\pi_n(X)$ by the usual action of the fundamental group in $X$.

Our main result is stated as follows:

THEOREM 1.3. *Assume that a connected CW-complex X satisfies*

$$\pi_i(X) = 0 \ (i > N) \quad \textit{or} \quad \dim X = N, \qquad \textit{for some integer } N,$$

*and that the cohomology groups of* (1.2) *are*

$$H^n(X_{n-1}; \pi_n(X)) = 0 \qquad (1 < n \leqq N).$$

*Then, the group* $\mathscr{E}_{\#}(X)$ *of* (1.1) *consists only of the identity* 1.