HIROSHIMA MATH. J.
5(1975), 181-185

Infinite Dimensional Noetherian Hilbert Domains

Kazunori FUJITA

(Received January 11, 1975)

1. As we know, examples of Hilbert domains are to a large extent the so-called equicodimensional rings, namely rings in which the height of every maximal ideal coincides with the dimension of R. As for a noetherian ring R, it is well known that R is an equicodimensional Hilbert ring if and only if the polynomial ring $R[X]$ in an indeterminate X over R is equicodimensional.

Examples of Hilbert domains with maximal ideals of different height were given by some authors (cf. [3], [6] and [7]). In particular, Heinzer constructed noetherian Hilbert domains with maximal ideals of various preassigned height. The purpose of this note is to give an alternative way of construction of such an example and to construct noetherian Hilbert regular domains with an infinite dimension.

The author wishes to express his hearty thanks to Prof. M. Nishi for his suggestions, and to Prof. H. Yanagihara for his comments in writing this paper.

2. All rings considered are commutative with identity. By a locally noetherian ring we mean a ring in which every localization by a maximal ideal is a noetherian ring, and for a prime ideal $\mathfrak{p}$ in a ring A, $depth(\mathfrak{p})$ means the dimension of $A/\mathfrak{p}$.

LEMMA 1. *Let A be a locally noetherian ring and $\mathfrak{p}$ be a prime ideal with $depth(\mathfrak{p})\geq 2$. Then the following statements a), b), and c) hold:*

a) $U=\{\mathfrak{P}\in Spec(A);\ \mathfrak{P}\supset\mathfrak{p},\ ht(\mathfrak{P}/\mathfrak{p})=1\}$ *is an infinite set.*

b) *If s is a non unit of A but not contained in $\mathfrak{p}$, then the subset of U consisting of $\mathfrak{P}$ which does not contain s is infinite.*

c) *Let U' be an infinite subset of U. Then we have* $\mathfrak{p}=\bigcap_{\mathfrak{P}\in U'}\mathfrak{P}$.

PROOF. If A is noetherian, it is clear that each statement holds. Otherwise, by taking a maximal ideal $\mathfrak{m}$ in A such that $\mathfrak{m}\supset\mathfrak{p}$ and $ht(\mathfrak{m}/\mathfrak{p})\geq 2$, it suffices to consider the ring $A_{\mathfrak{m}}$ in place of A.

LEMMA 2. *Let A be a locally noetherian ring. Then A is a Hilbert ring if and only if every prime ideal $\mathfrak{p}$ with $depth(\mathfrak{p})=1$ is the intersection of maximal ideals containing $\mathfrak{p}$.*

PROOF. The "only if" part is obvious. We assume that for every prime ideal $\mathfrak{p}$ with $depth(\mathfrak{p})=1$, $\mathfrak{p}$ is the intersection of maximal ideals containing $\mathfrak{p}$. As