HIROSHIMA MATH. J.
5 (1975), 135–139

Nonoscillation in Linear Second Order Ordinary Differential Equations

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(Received July 5, 1974)

Let q be a continuous function from $[0, \infty)$ to $(0, \infty)$, and consider

$$u''+qu = 0 \tag{1}$$

on $[0, \infty)$. It has long been known that if (1) is nonoscillatory then

$$\int_0^\infty q(t)dt < \infty \tag{2}$$

and

$$\limsup_{t\to\infty} t\int_t^\infty q(s)ds \leq 1 \tag{3}$$

(see [1] and [2], also [3, Chapter 2]). From (3) it is clear that if (1) is nonoscillatory then

$$\int_0^\infty\left(\int_t^\infty q(s)ds\right)^2 dt < \infty . \tag{4}$$

Under the assumption that (1) is nonoscillatory we shall obtain a result which shows that (2), (3), and (4) can be extended to

$$\limsup_{t\to\infty} t\left(\int_t^\infty q(s)ds+\int_t^\infty\left(\int_s^\infty q(\xi)d\xi\right)^2 ds\right) \leq 1 , \tag{5}$$

$$\int_0^\infty q(t)\exp\left(\int_0^t sq(s)ds\right)dt < \infty , \tag{6}$$

and

$$\int_0^\infty\left(\int_t^\infty q(s)ds\right)^2 \exp\left(\int_0^t sq(s)ds\right)dt < \infty . \tag{7}$$

It is clear that (5) is an extension of (3), and since nonoscillation does not imply

$$\int_0^\infty tq(t)dt < \infty ,$$

(6) and (7) are extensions of (2) and (4).