

A Remark on Duality of Domination Principle

Makoto OHTSUKA

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In this paper we remark that we obtain a simpler proof of Higuchi's fundamental theorem (Theorem 1) in [1] if we treat four kernels. We stress that (a) and (b) in our theorem are mutually dual.

We shall use the same notation as in [1]. As the space X with which we are concerned we take a locally compact Hausdorff space satisfying the second axiom of countability as in [1].

THEOREM. *Let G, K, L and N be lower semi-continuous function-kernels on $X \times X$. Then the following statements are equivalent:*

(a) *For $\mu \in E_0$ and $\nu \in M_0$, the inequality $G\mu \leq K\nu$ on $S\mu$ implies the inequality $L\mu \leq N\nu$ on X .*

(b) *For $\mu \in E_0$ and $\nu \in M_0$, the inequality $\check{G}\mu \leq \check{L}\nu$ on $S\mu$ implies the inequality $\check{K}\mu \leq \check{N}\nu$ on X .*

To prove this theorem we need

LEMMA (cf. Lemmas 3 and 4 in [1]). *Let G, K, L and N be as above. Suppose that for any $\mu \in C_{\text{supp}}(G)$ and $\nu \in M_0$, the inequality $G\mu \leq K\nu$ on $S\mu$ implies the inequality $L\mu \leq N\nu$ on X . Then, for any $\mu \in E_0$ and $\nu \in M_0$, $G\mu \leq K\nu$ on $S\mu$ implies $L\mu \leq N\nu$ on X .*

PROOF. Choose $\{\mu_n\}$ as in Lemma 2 of [1]. We have $G\mu_n \leq G\mu \leq K\nu$ on $S\mu_n \subset S\mu$. By our assumption $L\mu_n \leq N\nu$ holds on X . Therefore $L\mu \leq \varinjlim_{n \rightarrow \infty} L\mu_n \leq N\nu$ on X . This proves our lemma.

The following proof of our theorem is analogous to that of Theorem 1 in [1]. We need only to prove (a) \rightarrow (b). Let us assume (a). In view of our lemma it suffices to conclude $\check{K}\mu \leq \check{N}\nu$ on X for $\mu \in C_{\text{supp}}(G)$ and $\nu \in M_0$, assuming that $\check{G}\mu \leq \check{L}\nu$ holds on $S\mu$. Take a sequence $\{G_n\}$ (resp. $\{K_n\}$) of finite continuous function-kernels on $X \times X$ increasing to G (resp. K). As in [1] there exists a sequence $\{\lambda_{n,p}\}_{p=1}^{\infty}$ of positive measures supported by $S\mu$ such that

$$\begin{aligned} G_p \lambda_{n,p} &\geq K_n \varepsilon_y && \text{on } S\mu, \\ G_p \lambda_{n,p} &= K_n \varepsilon_y && \text{on } S\lambda_{n,p}, \end{aligned}$$

where ε_y means the unit measure at y . Let $\lambda_n \in E_0(G)$ be a vague adherent point