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Duality of Domination Principle for Non-symmetric Lower Semi-continuous Function-kernels

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1. Introduction

Let G be a lower semi-continuous function-kernel on the product space of a locally compact Hausdorff space X, and $\check{G}$ be the adjoint kernel of G defined by $\check{G}(x, y)=G(y, x)$. We say that the duality of the domination principle holds for G when the following statement is true: $\check{G}$ satisfies the domination principle if and only if G does.

M. Kishi first proved in [5] that the duality of the domination principle holds for a lower semi-continuous function-kernel G under the additional condition that G and $\check{G}$ satisfy the continuity principle.

For a continuous (in the extended sense) function-kernel G, M. Itô and the author verified in [2] that both G and $\check{G}$ satisfy the continuity principle when G satisfies the domination principle (cf. [3]). From this fact, follows the duality of the domination principle for continuous function-kernels.

Concerning a lower semi-continuous (but not continuous) function-kernel, the domination principle does not imply the continuity principle. So the analogous argument does not hold.

Let G and N be lower semi-continuous function-kernels on X. In this paper, we shall verify that G satisfies the relative domination principle with respect to N if and only if $\check{G}$ satisfies the transitive domination principle with respect to $\check{N}$. This was first obtained by M. Kishi in [5] under the assumption that G and $\check{G}$ satisfy the continuity principle. For continuous function-kernels, the author [1] can avoid this additional condition.

The duality of the domination principle follows immediately from the above equivalence.

As applications of these results, we shall investigate the relations among the potential theoretical principles and shall show the transitive law for the relative domination principle.

2. Notation

Let X be a locally compact Hausdorff space satisfying the second axiom of