HIROSHIMA MATH. J.
5 (1975), 525-535

Weak Solutions for Certain Nonlinear Time-dependent Parabolic Variational Inequalities

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(Received May 20, 1975)

1. Introduction

For a (real) Banach space V, in general, we denote by V^* the dual space of V, by $\|\cdot\|_V$ and $\|\cdot\|_{V*}$ the norms in V and V^*, respectively, and by $(\cdot, \cdot)_V$ the natural pairing between V^* and V.

Let A be a (multivalued) operator from a Banach space V into V^*, that is, to each $v \in V$ a subset Av of V^* be assigned. Then we define

$$D(A) = \{v \in V; Av \neq \phi\},$$

$$R(A) = \bigcup_{v \in V} Av$$

and

$$G(A) = \{[v, v^*] \in V \times V^*; v \in D(A), v^* \in Av\},$$

which are called the domain, the range and the graph of A, respectively. An operator A: $V \to V^*$ is called monotone if

$$(v^* - w^*, v - w)_V \geq 0 \quad \text{for any } [v, v^*], [w, w^*] \in G(A).$$

If A is monotone and there is no proper monotone extension of A, then A is called maximal monotone.

As an important class of maximal monotone operators from a Banach space V into V^*, there is a class of duality mappings. Let μ be a continuous strictly increasing function from $[0, \infty)$ into itself such that $\mu(0) = 0$ and $\mu(r) \uparrow \infty$ as $r \uparrow \infty$. The mapping $\mathscr{F}_\mu$: $V \to V^*$ defined by

$$\mathscr{F}_\mu(v) = \{v^* \in V^*;\ (v^*,\ v)_V = \mu(\|v\|_V)\|v\|_V \text{ and } \|v^*\|_{V*} = \mu(\|v\|_V)\}$$

is called the duality mapping of V into V^* associated with the gauge function μ. We know (cf. [6; Chapter 1]) that any duality mapping is singlevalued and demicontinuous (i.e., continuous with respect to the strong topology of V and the weak topology of V^*) provided that V is reflexive and V^* is strictly convex. Also, it is well-known (cf. [16; Proposition 1]) that a monotone operator A: $V \to V^*$ is maximal monotone if and only if the sum of A and at least one duality