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On Flat Extensions of Krull Domains

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Let A and B be Krull domains with A contained in B. We say that the condition "*no blowing up*", abbreviated to NBU, is satisfied if $ht(\mathfrak{P} \cap A) \leqq 1$ for every divisorial prime ideal $\mathfrak{P}$ of B. The main purpose of this paper is to give a criterion of the condition NBU by making use of the notion of divisorial modules, which was introduced in [5]. That is, the condition NBU is satisfied for Krull domains A and B if and only if B is divisorial as an A-module (Theorem 1). As an immediate consequence of the above criterion, we can obtain the well-known theorem: If B is flat over A, then the condition NBU is satisfied.

We shall also investigate the behavior of divisorial envelope under flat extensions of Krull domains. The main result is stated as follows: If, in addition to flatness, B is integral over A, $M \otimes B$ is a divisorial B-module for any codivisorial and divisorial A-module M.

We shall use freely the notation and the terminologies of [5] and [6].

§1. Flat modules over a Krull domain

In this section, we understand that A is always a Krull domain and K is the quotient field of A.

It is known that an A-lattice M is divisorial if and only if every regular A-sequence of length two is a regular M-sequence (cf. [4], Chap. I, § 5, Coroll. 5.5. (f)). This result is valid for any torsion free divisorial module and to prove this, a similar method can be applied. Namely we have

PROPOSITION 1. *Let M be a torsion-free A-module. Then M is divisorial if and only if every regular A-sequence of length two is a regular M-sequence.*

The following corollary is a direct consequence of Prop. 1.

COROLLARY. *If M is a flat A-module, then M is divisorial.*

PROPOSITION 2. *Let M be an A-module and N be a flat A-module. Then we have:*

(i) *If M is codivisorial, then so is $M \otimes_A N$.*

(ii) $\tilde{M} \otimes_A N = \widetilde{M \otimes_A N}$.