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(λ, μ)-*Absolutely Summing Operators*

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Introduction

Pietsch [9] introduced the concept of absolutely p-summing operators in normed spaces. This concept was extended in Ramanujan [10] to absolutely λ-summing operators by the aid of symmetric sequence spaces λ. On the other hand, Mityagin and Pelczyński [6] introduced the concept of (p, r)-absolutely summing operators in Banach spaces and this was recently extended in Miyazaki [7] to $(p, q; r)$-absolutely summing operators by using the sequence spaces $l_{p,q}$ and l_r. The object of this paper is to extend these two kinds of concepts to (λ, μ)-absolutely summing operators in normed spaces by making use of abstract sequence spaces λ and μ and to develop a theory of such operators.

In Section 1, we define the sequence spaces λ of type Λ and the sequence spaces μ of type M and define the (λ, μ)-absolutely summing operators. It is shown that $l_{p,q}$ is a space of type Λ and l_r is a space of type M. In Section 2, we state some basic properties of (λ, μ)-absolutely summing operators. We investigate in Section 3 some inclusion relations between the spaces of (λ_1, μ_1)- and (λ_2, μ_2)-absolutely summing operators. Section 4 is devoted to studying composition of two (λ, μ)-absolutely summing operators. Two spaces of (λ_1, μ_1)- and (λ_2, μ_2)-absolutely summing operators may happen to coincide, when their domain and range are particular normed spaces. These facts will be investigated in Section 5.

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§1. Notations and Definitions

For a sequence space λ the α-dual is denoted by $\lambda^{\times}$. If $\lambda^{\times\times} = \lambda$, then λ is said to be a perfect sequence or a Köthe space. We start with the sequence space c_0 of all scalar sequences converging to zero and the sequence space ω of all scalar sequences, which are given respectively an extended quasi-norm p and an extended norm q satisfying the following conditions:

(a) *If for any* $x = (x_1, \dots, x_n, \dots) \in c_0$ *and* $y = (y_1, \dots, y_n, \dots) \in \omega$ *we set* $x^i = (x_1, \dots, x_i, 0, \dots)$ *and* $y^i = (y_1, \dots, y_i, 0, \dots)$ *for* $i = 1, 2, \dots,$ *then* $p(x^i) \to p(x)$ *and* $q(y^i) \to q(y)$.