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On the Boltzmann Equation for Rough Spherical Molecules with Internal Energy

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§1. Introduction, notations and the results

Consider a system of the same kind of a large number of (say, N) rough elastic spherical molecules with the mass m, the radius a, and the moment of inertia $\boldsymbol{I}=ma^2\kappa$ $(0\leq\kappa\leq 2/3)$. Such a molecule possesses energy of rotation which is interconvertible with energy of translation. This model was treated by F. B. Pidduck [5] (see also [3]). The motion of a molecule can be specified by a pair (ξ, α), where $\xi\in R^3$ is the velocity of its center and $\alpha\in R^3$ is its angular velocity. The dynamics of a collision is given in the following way: Let (ξ, α) and (η, β) be the velocity pairs after collision of two molecules (ξ', α') and (η', β'). Then,

$$(1.1)\qquad \begin{cases} \xi' = \xi+\dfrac{\kappa V+(\eta-\xi,\ \ell)\ell}{\kappa+1}, \\[2ex] \eta' = \eta-\dfrac{\kappa V+(\eta-\xi,\ \ell)\ell}{\kappa+1}, \\[2ex] \alpha' = \alpha+\dfrac{\ell\times V}{a(\kappa+1)}, \\[2ex] \beta' = \beta+\dfrac{\ell\times V}{a(\kappa+1)}, \end{cases}$$

where ℓ denotes the unit vector in the direction of the line from the center of the molecule (ξ, α) to that of the molecule (η, β) at collision and V the relative velocity, after impact, of the points of the spheres which come into contact, that is,

$$(1.2)\qquad V=\eta-\xi+a\ \ell\times(\alpha+\beta).$$

(In (1.1) and (1.2), (,) and $\times$ denote the inner and the outer product respectively.) Note that this dynamics preserves the linear momentum and the total energy, i.e.,

$$(1.3)\qquad \begin{cases} \xi'+\eta' = \xi+\eta, \\ [\xi', \alpha']^2+[\eta', \beta']^2 = [\xi, \alpha]^2+[\eta, \beta]^2, \end{cases}$$

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