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Locally Ascendantly Coalescent Classes of Lie Algebras

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Introduction

In the recent study of infinite-dimensional Lie algebras the concepts of coalescence, ascendant coalescence and local coalescence have been investigated considerably in detail [1–12]. However, very little is known concerning local ascendant coalescence. The only fact known about it is that the class $\mathfrak{N}$ of nilpotent Lie algebras over a field of characteristic 0 is locally ascendantly coalescent [5, 6]. Therefore it is desirable for us to know more about this concept. The purpose of this paper is to investigate the properties of locally ascendantly coalescent classes and to show that several known classes and others are locally ascendantly coalescent.

We shall show that if a class $\mathfrak{X}$ is a Q-closed and locally ascendantly coalescent (resp. ascendantly coalescent) subclass of $(\mathrm{E}\mathfrak{A})_{(\omega)}$ then the class $\mathfrak{X}_{(\omega)}$ is locally ascendantly coalescent (resp. ascendantly coalescent). We shall also show that, for the classes $\mathfrak{X}$ and $\mathfrak{Y}$ such that $\mathfrak{X} \leq \mathfrak{Y} \leq \acute{\mathrm{M}}\mathfrak{X}$, $\mathfrak{X}$ is locally ascendantly coalescent if and only if so is $\mathfrak{Y}$. The applications of these results to the special case where $\mathfrak{X}$ is the class $\mathfrak{N}$ of nilpotent Lie algebras over a field of characteristic 0 yield that the following classes are locally ascendantly coalescent: $\mathfrak{N}_{(\omega)}$, the class $\mathfrak{D}$ (resp. $\mathfrak{D}'$) of Lie algebras L such that every subalgebra of L is a subideal (resp. an ascendant subalgebra), the class $\mathfrak{Fi}$ of Fitting algebras, the class $\mathfrak{B}$ of Baer algebras, the class $\mathfrak{Gr}$ of Gruenberg algebras, and the class $\mathfrak{Z}$ of hypercentral algebras.

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Throughout this paper, we shall be concerned with Lie algebras over an arbitrary field Φ which are not necessarily finite-dimensional, and we denote by $\mathfrak{X}$ an arbitrary class of Lie algebras over Φ, unless otherwise specified.

$\mathfrak{X}$ is ascendantly coalescent provided the join of two ascendant $\mathfrak{X}$-subalgebras of any Lie algebra L is always an ascendant $\mathfrak{X}$-subalgebra of L. $\mathfrak{X}$ is locally ascendantly coalescent [5] provided whenever H and K are ascendant $\mathfrak{X}$-subalgebras of a Lie algebra L, for every finite subset F of the join $\langle H, K\rangle$ there exists an ascendant $\mathfrak{X}$-subalgebra X of L such that $F \subseteq X \leq \langle H, K\rangle$. Any ascendantly coalescent class of Lie algebras is obviously locally ascendantly coalescent.

We denote by $\mathfrak{N}$, $\mathrm{E}\mathfrak{A}$, $\mathfrak{F}$ and $\mathfrak{G}$ respectively the classes of nilpotent, solvable,