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On the Existence of Boundary Values of Beppo Levi Functions Defined in the Upper Half Space of R^n

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1. Introduction and statement of results

Let R^n ($n \geqq 2$) be the n-dimensional Euclidean space and its points be denoted by x, y, etc., or $x=(x_1, x_2,\dots, x_n)=(x', x_n)$, $y=(y_1, y_2,\dots, y_n)=(y', y_n)$, etc. For a positive number α such that $\alpha<n$, the Riesz potential of order α of a measure μ on R^n is defined by

$$U_\alpha^\mu(x) = \int |x-y|^{\alpha-n} d\mu(y).$$

If μ has a density f (that is, $d\mu=fdx$, where f is locally integrable), we may write U_α^f instead of U_α^μ. The Riesz capacity $C_\alpha(E)$ of a Borel set E in R^n may be defined as follows:

$$C_\alpha(E) = \sup \mu(R^n),$$

where the supremum is taken over all positive measures μ concentrated on E such that $U_\alpha^\mu(x) \leqq 1$ for every $x \in S_\mu$ (S_μ is the support of μ).

Our main theorem is the following:

THEOREM 1. *Let α and p be numbers such that $\alpha \geqq 0$ and $1+\alpha<p<n+\alpha$. Let f be a function which is defined and continuous in the upper half space $R_+^n=\{x=(x', x_n); x_n>0\}$. Suppose that all partial derivatives of f of first order exist a.e. on R_+^n and that for any bounded open set Ω in R_+^n*

$$\text{(1)} \qquad \iint_\Omega |\mathrm{grad} f(x', x_n)|^p x_n^\alpha dx' dx_n < \infty.$$

Then $\lim_{x_n \downarrow 0} f(x', x_n)$ exists and is finite except for $(x', 0)$ in a Borel set E in $R_0^n=\{(y', 0); y' \in R^{n-1}\}$ such that $C_{p-\alpha}(E)=0$ if $p \leqq 2$ and $C_{p-\alpha-\varepsilon}(E)=0$ for any $\varepsilon>0$ with $p-\alpha-\varepsilon>0$ if $p>2$.

In the case $p=2$ this theorem was shown by H. Wallin [7]. He also showed that his result is the best possible as to the size of the exceptional set. We shall generalize this result in the following theorem:

THEOREM 2. *Let α and p be as in* Theorem 1. *Let E be a set in R_0^n such*