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On the Primary Decomposition of Differential Ideals

Shizuka SATÔ

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Let R be a commutative ring containing an identity. Then a derivation on R is an abelian group homomorphism $D: R \to R$ such that for all a, b in R,

$$D(ab) = aD(b) + bD(a).$$

A higher derivation of rank m on R is a sequence of abelian group homomorphisms $\delta_i: R \to R$, $i=0, 1, \dots, m$ such that

(1) δ_0 is the identity mapping,

(2) for all x, y in R and $i \geqq 1$,

$$\delta_i(xy) = \sum_{j+k=i} \delta_j(x)\delta_k(y).$$

We shall let $Der(R)$ denote the collection of all derivations on R and let $H_m(R)$ denote the collection of all higher derivations of rank m on R. If m is infinite, we shall call this sequence merely a higher derivation. Let $\mathfrak{a}$ be an ideal of R and let T, G be subsets of $Der(R)$, $H_m(R)$ respectively. We shall say that $\mathfrak{a}$ is a T-ideal if $D(\mathfrak{a}) \subseteqq \mathfrak{a}$ for all $D \in T$. Similarly, we shall say that $\mathfrak{a}$ is a G-ideal if for all $\delta = \{\delta_i\} \in G$, $\delta_i(\mathfrak{a}) \subseteqq \mathfrak{a}$ for all $i=0, 1, \dots, m$. Let x be an element of R. We shall denote by $[x]$ the smallest G-ideal that contains the element x. This is well defined since the intersection of any number of G-ideals is again a G-ideal.

On the primary decomposition of differential ideals, the following is known:

THEOREM A (Theorem 1, [1]). *Let R be a Noetherian ring and let $\mathfrak{a}$ be an ideal of R with associated prime ideals $\mathfrak{p}_1, \mathfrak{p}_2, \dots, \mathfrak{p}_s$. Let $\delta = \{\delta_i\}$ be a higher derivation such that $\mathfrak{a}$ is a δ-ideal. Then $\mathfrak{p}_1, \mathfrak{p}_2, \dots, \mathfrak{p}_s$ are also δ-ideals and $\mathfrak{a}$ can be written as an irredundant intersection $\mathfrak{q}_1 \cap \cdots \cap \mathfrak{q}_s$ of primary δ-ideals $\mathfrak{q}_i$.*

In this short paper, we wish to generalize Theorem A to the case of a set of higher derivations of rank m. Since in [1] they use essentially the fact that the rank is infinite, the method can not be used for the case of finite rank. So, we shall take up new techniques. We shall begin with the definition of G-primary G-ideals.

DEFINITION 1. *Let $\mathfrak{q}$ be a G-ideal of R. $\mathfrak{q}$ is called a G-primary G-ideal*