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Oscillatory and Asymptotic Behavior of Differential Equations with Deviating Arguments(*)

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1. Introduction

In this paper we are concerned with the oscillatory and asymptotic behavior of the n-th order $(n>1)$ nonlinear differential equation with deviating arguments of the form

$$(*)\qquad x^{(n)}(t)+\left\{\prod_{j=1}^{m_0}|x[\tau_{0j}(t)]|^{\rho_j}\right\}F(t;\ [x]^2<\tau_0(t)>,\ [x']^2<\tau_1(t)>,\dots$$
$$\dots,\ [x^{(n-1)}]^2<\tau_{n-1}(t)>)\prod_{j=1}^{2\lambda-1}\operatorname{sgn}x[\tau_{0j}(t)]=0,\qquad t\geqq t_0,$$

where λ is a positive integer so that $2\lambda-1\leqq m_0$ and:

$$(\forall j=1,2,\dots,m_0)\rho_j\geqq 0,$$

$$\sum_{j=1}^{m_0}\rho_j=1,$$

$$\tau_i(t)=(\tau_{i1}(t),\ \tau_{i2}(t),\dots,\ \tau_{im_i}(t)),$$

$$h<\sigma(t)>\ =(h[\sigma_1(t)],\ h[\sigma_2(t)],\dots,\ h[\sigma_m(t)]),\ \sigma=(\sigma_1,\ \sigma_2,\dots,\ \sigma_m).$$

In the particular case, where

$$(\forall i,\ j)\tau_{ij}(t)\equiv t$$

the above differential equation (*) becomes an ordinary differential equation.

For the real valued functions τ_{ij} $(j=1,2,\dots,m_i,\ i=0,1,\dots,n-1)$ and F we suppose that:

(i) The functions τ_{ij} are continuous on the half-line $[t_0,\infty)$ and

$$\lim_{t\to\infty}\tau_{ij}(t)=\infty.$$

(ii) F is non-negative on $[t_0,\infty)\times E_0$ and $\left(\prod_{j=1}^{m_0}y_{0j}^{e_j/2}\right)F(t;y_0,y_1,\dots,y_{n-1})$ is continuous on the same set, where $E_0=[0,\infty)^{m_0}\times[0,\infty)^{m_1}\times\cdots\times[0,\infty)^{m_{n-1}}$.

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