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On the Limits of p-Precise Functions along Lines Parallel to the Coordinate Axes of R^n

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1. Introduction and statement of the main result

Recently, C. Fefferman [2] proved the following result: Let $1<p<n$ and let u be a C^1-function on $R^n = R \times R^{n-1}$ $(n \geqq 2)$ such that $\int_{R^n} |\mathrm{grad}\, u|^p dx < \infty$. Then there is a constant c such that $\lim_{x_1 \to \infty} u(x_1, x') = c$ for almost all $x' \in R^{n-1}$.

In the present note, we shall give an improvement of this result by using the capacity $C_{1,p}$:

$$C_{1,p}(E) = \inf \|f\|_p^p \qquad \text{for} \quad E \subset R^n,$$

where the infimum is taken over all non-negative functions f in $L^p(R^n)$ such that $\int |x-y|^{1-n} f(y) dy \geqq 1$ for all $x \in E$. This capacity is a special case of the capacity $C_{k;\mu;p}$ introduced by N. G. Meyers [4]. We shall show

THEOREM 1. *Let $1<p<n$ and let u be a p-precise function on $R^n = R \times R^{n-1}$. Then there are a constant c and a Borel set E' in R^{n-1} with $C_{1,p}(\{0\} \times E') = 0$ such that*

$$\lim_{x_1 \to \infty} u(x_1, x') = c \qquad \text{for all} \quad x' \in R^{n-1} - E'.$$

For p-precise functions, see [6; Chap. IV] (also cf. [3; Chap. III, §2], in which they are called Beppo Levi functions of order p). Note that for a p-precise function u on R^n, $\mathrm{grad}\, u$ is defined almost everywhere and $\int_{R^n} |\mathrm{grad}\, u|^p dx < \infty$. Also note that if $C_{1,p}(\{0\} \times E') = 0$, then the $(n-1)$-dimensional Lebesgue measure of E' is zero (see [3; Theorem A], [1; Theorem 1 in §IV] and our Lemma 2).

The proof of this theorem is based on the following proposition, which is a special case of Theorem 1 on account of [6; Theorem 9.6] (also cf. [5; Theorem 5.1]).

PROPOSITION 1. *Let $1<p<n$ and let $f \in L^p(R^n)$. Then there is a Borel set $E' \subset R^{n-1}$ with $C_{1,p}(\{0\} \times E') = 0$ such that*