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Holomorphic Mappings into Closed Riemann Surfaces

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1. Introduction

Let X be an irreducible normal complex space which is a k-sheeted (ramified) covering space over the m-dimensional complex affine space C^m and $L \to V$ a positive holomorphic line bundle over a smooth projective variety of dimension $n \leqq m$. In [9] we proved the following defect relation:

Let $f: X \to V$ be a non-degenerate meromorphic mapping. Then for divisors D_i $(i=1,\dots, q)$ determined by global holomorphic sections of L such that $\sum_{i=1}^{q} D_i$ has simple normal crossings,

$$\sum_{i=1}^{q} \delta(D_i) \leqq \left[\frac{c(K_V^{-1})}{c(L)}\right] + 2(k-1)l_0, \tag{1.1}$$

where K_V denotes the canonical bundle over V and l_0 is an integer independent of each $\{D_i\}$ (cf. (2.6) and Theorem B in section 2).

We wish to investigate what the defect relation (1.1) amounts to, determining its right hand side more explicitly in the rather simple case $\dim X = \dim V = 1$. This is the first aim of the present note.

Since R. Nevanlinna created his theory of meromorphic functions in the complex plane C (cf. [6]), many authors have done its generalization for holomorphic mappings between two abstract Riemann surfaces R and S in various ways. Sario [11] obtained a very general defect relation and simultaneously showed that if the genus of S is greater than 1, there can be only a few restricted cases where there really exist non-trivial (= non-constant) holomorphic mappings f: $R \to S$ for which the general defect relation remains valid in its proper sense. Therefore it is one of the most interesting problems to determine the types (in any sense) of the Riemann surfaces R and S admitting non-trivial holomorphic mappings from R into S for which the defect relation holds in the proper sense. So far as the existence of non-trivial holomorphic mappings from R into S, Ozawa [10], Mutó [5], Hiromi-Mutō [3] and Niino [7, 8] dealt with this problem in the case when R (resp. S) is a finitely sheeted covering surface over C (resp. C or the Riemann sphere P^1). Our second aim is to study holomorphic mappings f: $X \to S$, where X is a finitely sheeted covering surface over C and S is a closed Riemann surface, from a point of view different from that of Ozawa [10], Mutó [5] and Hiromi-Mutó [3]. In the case when X is a 2-sheeted covering surface over C, we shall